

# Relativistic interpretation of the nature of the nuclear tensor force<sup>\*</sup>

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**Abstract:** The spin-dependent nature of the nuclear tensor force is studied in detail within the relativistic Hartree-Fock approach. The relativistic formalism for the tensor force is supplemented with an additional Lorentz-invariant tensor formalism in the  $\sigma$ -scalar channel, so as to take into account almost fully the nature of the tensor force brought about by the Fock diagrams in realistic nuclei. Specifically, the tensor sum rules are tested for the spin and pseudo-spin partners with and without nodes, to further understand the nature of the tensor force within the relativistic model. It is shown that the interference between the two components of nucleon spinors causes distinct violations of the tensor sum rules in realistic nuclei, mainly due to the opposite signs on the  $\kappa$  quantities of the upper and lower components, as well as the nodal difference. However, the sum rules can be precisely reproduced if the same radial wave functions are taken for the spin/pseudo-spin partners in addition to neglecting the lower/upper components, revealing clearly the nature of the tensor force.

**Keywords:** nuclear tensor force, sum rule, covariant density functional theory, relativistic Hartree-Fock theory

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## 1 Introduction

In nuclear physics, the nuclear force that binds protons and neutrons into atomic nuclei is one of the most important issues, and many attempts have been devoted to explaining its nature. The meson exchange diagram of the nuclear force proposed by Yukawa is still one of the most successful attempts [1]. In recent years, the worldwide construction of a new generation of radioactive ion beam facilities has greatly promoted the development of the field and a set of novel phenomena have been discovered in exotic nuclei. This brings a series of challenges and opportunities for nuclear physics, especially in understanding the nature of the nuclear force. A typical example is the non-central tensor force, which has drawn considerable attention due to its characteristic spin-dependent nature [2]. It plays an essential role in determining nuclear shell evolution [3–17], nuclear isospin excitations and  $\beta$ -decays [18–23], and the properties of nuclear matter [24].

Based on the meson exchange diagram of nuclear force, the relativistic description of nuclear structure properties has achieved great progress in combination with density functional theory, namely the famous covariant density functional (CDF) theory (see Ref. [25] and references therein). During the past decades, relativistic mean field (RMF) theory, the CDF theory with

out Fock terms, has received more and more attention due to its successful description of many nuclear phenomena in both stable and exotic nuclei [26–37]. With the covariant frame, mainly represented as large scalar and vector fields of the order of a few hundred MeV, the RMF theory can provide a self-consistent description of the spin-orbit (SO) couplings, an important ingredient of nuclear force. While limited by the RMF approach itself, the important degrees of freedom in the meson exchange diagram, such as the  $\pi$ - and  $\rho$ -tensor couplings, are missing, and specifically the important ingredient of nuclear force — the tensor force, arising from the  $\pi$  exchange and  $\rho$ -tensor coupling, cannot be efficiently taken into account. The  $\pi$ - and  $\rho$ -tensor couplings, as well as the tensor force, can be considered only (or mainly) with the presence of Fock terms, which are in general ignored in the RMF scheme for simplicity.

Because of the complexity induced by Fock terms and the limitation of computer power, providing an appropriate quantitative description of nuclear structure properties under the relativistic Hartree-Fock (RHF) approach remains a long-standing problem [38–42]. Until ten years ago, a new RHF approach, namely the density-dependent relativistic Hartree-Fock (DDRHF) theory [43–45], also referred as the CDF theory with Fock terms, was developed in combination with the density-dependent meson-nucleon coupling, and a quantitative

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description of nuclear structure properties was achieved with comparable accuracy to the standard CDF models. Due to Lorentz covariance, the RHF approach maintains the advantages of the RMF, i.e., the self-consistent treatment of the spin-orbit coupling. Moreover, the presence of Fock terms has brought significant improvements in describing nuclear properties, such as self-consistent description of shell evolution [8, 9, 16, 46, 47], better preserved pseudo-spin and spin symmetries [44, 48–51], and fully self-consistent treatment of nuclear isospin excitation [52–54] and decay modes [55, 56]. Besides, the Fock terms also present distinct contributions to nuclear symmetry energy [57–59].

Recently, analysis within the DDRHF theory has shown that the Fock terms of the meson-nucleon couplings represent distinct spin dependence [60], a characteristic nature of the tensor force [2]. It was then recognized that the Fock diagrams of the meson-nucleon couplings can take the important ingredient of nuclear force — the tensor force — into account naturally [60]. Particularly, more remarkable tensor effects are found in the Fock terms of the isoscalar  $\sigma$ - $S$  and  $\omega$ - $V$  couplings, rather than the isovector  $\rho$ - $V$ ,  $\rho$ - $T$  and  $\pi$ - $PV$  couplings. In Ref. [60], a series of relativistic formalisms have been proposed for the tensor force components in the Fock diagrams of various meson-nucleons couplings, and the self-consistent tensor effects were analysed for nuclear ground states and nuclear matter with the proposed relativistic formalism [24, 60]. Without introducing any additional free parameters, the spin-dependent feature brought about by the Fock terms can be interpreted almost completely by the proposed relativistic formalism. In addition, the reduction of the kinetic part of symmetry energy at the supranuclear density region in the DDRHF theory can be regarded partly as the effect of the nuclear tensor force [59].

Notice that the tensor force, as derived from shell model calculations [2], fulfills some specific sum rules [see Eqs. (12) and (13)] quantitatively owing to its spin-dependent feature. Conceptually, the sum rules were verified in Ref. [60] under the assumption of neglecting the lower components of Dirac spinors and taking the same radial wave functions for the spin partner  $j_{\pm} = l \pm 1/2$  states. Since the lower and upper components of Dirac spinors are of different angular momenta, leading to opposite parity, the sum rules could be violated distinctly with the complete form of the Dirac spinors in realistic nuclei. To better understand the nature of the tensor force, it is worth testing the sum rule in realistic nuclei with the relativistic formalism of tensor forces, and to reveal the relativistic effect brought about by the lower components of Dirac spinors. The contents are organized as follows. In Section 2, the relativistic formalism for the tensor force components in the Fock diagrams of meson-

nucleon couplings are recalled and a supplementation to the coupling in the  $\sigma$ - $S$  coupling channel is presented. In Section 3 the sum rules are verified by taking the spin/pseudo-spin partners in  $^{48}\text{Ca}$ ,  $^{90}\text{Zr}$  and  $^{208}\text{Pb}$  as examples to understand the nature of the tensor force, and the contributions from the lower/upper components are discussed in detail. Finally, a brief summary is given in Section 4.

## 2 Supplementation of the relativistic formalism for tensor force components in $\sigma$ - $S$ channel

For completeness, the relativistic formalism for the tensor force components in the Fock terms of the  $\sigma$ - $S$ ,  $\omega$ - $V$ ,  $\rho$ - $V$ ,  $\rho$ - $T$  and  $\pi$ - $PV$  couplings are recalled as follows,

$$\mathcal{H}_{\sigma-S}^{T_1} = -\frac{1}{2} \cdot \frac{1}{2} \left[ \frac{g_{\sigma}}{m_{\sigma}} \bar{\psi} \gamma_0 \Sigma_{\mu} \psi \right]_1 \left[ \frac{g_{\sigma}}{m_{\sigma}} \bar{\psi} \gamma_0 \Sigma_{\nu} \psi \right]_2 D_{\sigma-S}^{T, \mu\nu}(1, 2), \quad (1)$$

$$\mathcal{H}_{\omega-V}^T = +\frac{1}{2} \cdot \frac{1}{2} \left[ \frac{g_{\omega}}{m_{\omega}} \bar{\psi} \gamma_{\lambda} \gamma_0 \Sigma_{\mu} \psi \right]_1 \left[ \frac{g_{\omega}}{m_{\omega}} \bar{\psi} \gamma_{\delta} \gamma_0 \Sigma_{\nu} \psi \right]_2 \times D_{\omega-V}^{T, \mu\nu\lambda\delta}(1, 2), \quad (2)$$

$$\mathcal{H}_{\rho-T}^T = +\frac{1}{2} \left[ \frac{f_{\rho}}{2M} \bar{\psi} \sigma_{\lambda\mu} \vec{\tau} \psi \right]_1 \cdot \left[ \frac{f_{\rho}}{2M} \bar{\psi} \sigma_{\delta\nu} \vec{\tau} \psi \right]_2 D_{\rho-T}^{T, \mu\nu\lambda\delta}(1, 2), \quad (3)$$

$$\mathcal{H}_{\pi-PV}^T = -\frac{1}{2} \left[ \frac{f_{\pi}}{m_{\pi}} \bar{\psi} \gamma_0 \Sigma_{\mu} \vec{\tau} \psi \right]_1 \cdot \left[ \frac{f_{\pi}}{m_{\pi}} \bar{\psi} \gamma_0 \Sigma_{\nu} \vec{\tau} \psi \right]_2 D_{\pi-PV}^{T, \mu\nu}(1, 2), \quad (4)$$

where the relativistic spin operator  $\Sigma^{\mu} = (\gamma^5, \mathbf{\Sigma})$ ,  $M$  is the nucleon mass, and  $\vec{\tau}$  denotes the isospin operator of the nucleon ( $\psi$ ). The propagator terms  $D^T$  read as,

$$D_{\phi}^{T, \mu\nu}(1, 2) = \left[ \partial^{\mu}(1) \partial^{\nu}(2) - \frac{1}{3} g^{\mu\nu} m_{\phi}^2 \right] D_{\phi}(1, 2) + \frac{1}{3} g^{\mu\nu} \delta(x_1 - x_2), \quad (5)$$

$$D_{\phi'}^{T, \mu\nu\lambda\delta}(1, 2) = \partial^{\mu}(1) \partial^{\nu}(2) g^{\lambda\delta} D_{\phi'}(1, 2) - \frac{1}{3} \left( g^{\mu\nu} g^{\lambda\delta} - \frac{1}{3} g^{\mu\lambda} g^{\nu\delta} \right) m_{\phi'}^2 D_{\phi'}(1, 2) + \frac{1}{3} \left( g^{\mu\nu} g^{\lambda\delta} - \frac{1}{3} g^{\mu\lambda} g^{\nu\delta} \right) \delta(x_1 - x_2), \quad (6)$$

where  $\phi$  stands for the  $\sigma$ - $S$  and  $\pi$ - $PV$  channels, and  $\phi'$  represents the  $\omega$ - $V$  and  $\rho$ - $T$  channels. Here to distinguish the terms, we use  $\mathcal{H}_{\sigma-S}^{T_1}$  to denote the relativistic formalism for tensor force components in the  $\sigma$ - $S$  channel proposed by Ref. [60]. For the  $\rho$ - $V$  channel, the corresponding formalism  $\mathcal{H}_{\rho-V}^T$  can be obtained simply by replacing  $m_{\omega}(g_{\omega})$  in Eqs. (2) and (6) by  $m_{\rho}(g_{\rho})$  and inserting the isospin operator  $\vec{\tau}$  in the interacting index.

In the prior study on the tensor effects brought about

by the Fock terms, the lower components of Dirac spinors were dropped and the spin partners  $j_{\pm} = l \pm 1/2$  were assumed to share the same radial wave functions [60]. Naturally, it would be interesting to study the tensor effects with the complete form of the relativistic wave functions, namely for realistic nuclei. Utilizing the full Dirac spinors determined by the self-consistent calculations with the RHF-PKA1 model, we calculate the contributions from various single-particle orbits  $j'$  to the SO splitting of the spin-partner states  $j_{\pm}$ , defined as

$$\Delta E_{SO} \equiv V_{j_{-}j'} - V_{j_{+}j'}, \quad (7)$$

where  $V_{jj'}$  denotes the interaction matrix element for the single-particle states  $j$  and  $j'$ . In fact, not only the contributions from the direct terms but also those from the exchange parts of single-particle potentials give rise to the SO splitting in nuclear single-particle spectra. However, the spin-dependent feature of the contributions  $\Delta E_{SO}$  from the nucleon-nucleon interactions, namely the difference in values between the spin-partner states  $j'_{\pm} = l \pm 1/2$ , is dominated by the Fock diagrams, particularly via the isoscalar meson-nucleon coupling channels [60]. Such a spin dependence of  $\Delta E_{SO}$  is then well explained by introducing a series of Lorentz-invariant tensor formalisms (Eqs. (1)–(4)), and the corresponding tensor interaction matrix element reads as

$$V_{jj'}^T = \bar{f}_j(\mathbf{r}_1) \bar{f}_{j'}(\mathbf{r}_2) \Gamma_{1,2}^T f_j(\mathbf{r}_2) f_{j'}(\mathbf{r}_1), \quad (8)$$

with  $\Gamma_{1,2}^T$  the interaction vertices of the tensor force and the nucleon Dirac spinor

$$f_a(\mathbf{r}) = \frac{1}{r} \begin{pmatrix} iG_a(r) \\ F_a(r) \hat{\boldsymbol{\sigma}} \cdot \hat{\mathbf{r}} \end{pmatrix} \mathcal{Y}_a(\hat{\mathbf{r}}) \chi_{\frac{1}{2}}(q_a), \quad (9)$$

where  $\mathcal{Y}_a(\hat{\mathbf{r}})$  is the spinor spherical harmonic, and  $\chi_{\frac{1}{2}}(q_a)$  the isospinor.  $G_a$  and  $F_a$  correspond to the upper and lower components of the radial wave function, respectively. Thus,  $V_{jj'}^T$  can be divided further with respect to the  $G_a$  and  $F_a$  components of the states  $j$  and  $j'$ , namely,

$$V_{jj'}^T = V_{G_j G_{j'}}^T + V_{G_j F_{j'}}^T + V_{F_j G_{j'}}^T + V_{F_j F_{j'}}^T. \quad (10)$$

Figure 1 shows the contributions to  $\Delta E_{SO}$  only from the Fock diagrams, giving the total, the tensor parts and the central parts, respectively, and taking the nodeless neutron ( $\nu$ ) orbits in  $^{48}\text{Ca}$  as examples. From Figs. 1(b) and 1(c), it is found that the spin-dependent feature of  $\Delta E_{SO}$  in Fock terms can be almost fully interpreted by the relativistic formalism for the tensor force components in the  $\omega$ - $V$  channel [i.e., Eq. (2)], while it seems to be overestimated in the  $\sigma$ - $S$  coupling channel, implying that the spin-dependent feature introduced by the  $\sigma$ - $S$  coupling cannot be exactly explained by Eq. (1).

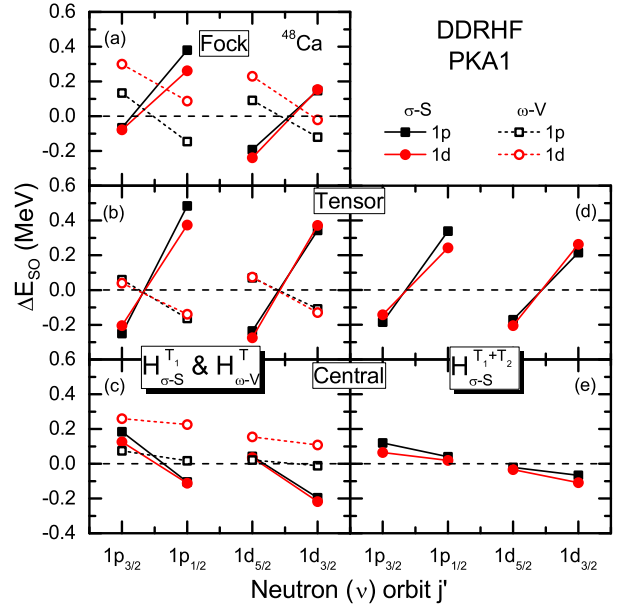


Fig. 1. (color online) Contributions to the spin-orbit (SO) splittings  $\Delta E_{SO} = V_{j_{-}j'} - V_{j_{+}j'}$  (MeV): the Fock terms [plot (a)], the tensor parts [plots (b) and (d)] and the central parts [plot (c) and (e)]. In plots (d-e), the supplemented tensor terms  $\mathcal{H}_{\sigma-S}^{T_2}$  are taken into account. The results are extracted from the calculations of DDRHF functional PKA1 for nodeless neutron ( $\nu$ ) orbits in  $^{48}\text{Ca}$ , and full Dirac spinors are utilized in calculating the interaction matrix element  $V_{jj'}$ .

The  $\pi$ - $PV$  coupling is originally in the form of Lorentz tensor couplings of rank 2 [38, 43], and it is proved that the spin-dependent feature introduced by the  $\pi$ - $PV$  coupling can be fully explained indeed by  $\mathcal{H}_{\pi-PV}^T$ , even without introducing the approximation on the radial wave functions. In fact, from the relativistic formalism of the tensor forces in Eqs. (1)–(4), all the formalism certainly represents the relativistic-type (Lorentz) tensor couplings but taking various ranks, formally at rank 4 for the  $\omega$ - $V$  and  $\rho$ - $T$  channels, though for the latter it is actually at rank 3, and rank 2 for the  $\sigma$ - $S$  and  $\pi$ - $PV$  channels. In order to avoid overestimation of the tensor effects involved by the Fock terms of the  $\sigma$ - $S$  coupling [see Figs. 1(b) and 1(c)], higher ranks of Lorentz tensor coupling than Eq. (1) could be considered as supplementations. The relativistic formalism of tensor force at rank 3, similar to  $\mathcal{H}_{\rho-T}^T$ , is naturally the first choice, i.e.,

$$\mathcal{H}_{\sigma-S}^{T_2} = +\frac{1}{4} \frac{1}{9} \left[ \frac{g_\sigma}{m_\sigma} \bar{\psi} \sigma_{\lambda\mu} \psi \right]_1 \cdot \left[ \frac{g_\sigma}{m_\sigma} \bar{\psi} \sigma_{\delta\nu} \psi \right]_2 D_{\sigma-S}^{T,\mu\nu\lambda\delta}(1,2), \quad (11)$$

where the propagator term  $D_{\sigma-S}^{T,\mu\nu\lambda\delta}$  corresponds to Eq. (6) with  $\phi' = \sigma$ - $S$ . As shown in Figs. 1(d) and 1(e), the supplemented relativistic formalism for the tensor force

components in  $\sigma$ - $S$  channel, i.e.,  $\mathcal{H}_{\sigma-S}^{T_1+T_2}$ , improves remarkably the description of the tensor nature involved by the Fock terms of  $\sigma$ - $S$  coupling, leading to a clear damping of the spin dependence of the central part (Figs. 1(e)) in comparison with that in the case of  $\mathcal{H}_{\sigma-S}^{T_1}$  (Figs. 1(c)). It is worth mentioning again that the full Dirac spinors determined self-consistently with the RHF-PKA1 model are utilized in calculating  $\Delta E_{\text{SO}}$ , different from Ref. [60]. For the other coupling channels, namely  $\omega$ - $V$ ,  $\rho$ - $V$ ,  $\rho$ - $T$  and  $\pi$ - $PV$ , it has been checked that the existing relativistic formalism of the tensor forces [i.e., Eqs. (2)–(4)] can describe fully the tensor effects involved by the relevant Fock diagrams even with the complete Dirac spinors.

As derived from the shell model calculations [2], the tensor force should fulfill the following sum rule,

$$V_{\text{sum}} \equiv \hat{j}_+^2 V_{j_+j'}^T + \hat{j}_-^2 V_{j_-j'}^T = 0, \quad (12)$$

where  $\hat{j}^2 = 2j+1$ . As mentioned in Refs. [2, 60], one has to choose the same radial wave functions for the spin partner states  $j_{\pm}$  to reproduce the sum rule exactly. In fact, the sum rule is often taken as the identity of the tensor force, which reveals clearly its spin-dependent nature. It is then worth seeing how precisely the sum rule is fulfilled by the relativistic formalism of tensor forces, even without introducing the approximation of the wave functions, for instance in a realistic nucleus. Particularly, as indicated by the supplemented relativistic formalism  $\mathcal{H}_{\sigma-S}^{T_1+T_2}$  for the  $\sigma$ - $S$  channels, it is also valuable to verify the role of the lower component of nucleon spinor, from which the relativistic effects are expected to be revealed.

### 3 Verification of tensor sum rule

In this section, the tensor sum rule is verified under the relativistic Hartree-Fock (RHF) approach with the functional PKA1. In order to provide a detailed understanding of the sum rule, spin-unsaturated magic systems, namely  $^{48}\text{Ca}$ ,  $^{90}\text{Zr}$  and  $^{208}\text{Pb}$ , are taken as examples in the following and the discussion focuses on the neutron spin partner states without and with nodes, and the pseudo-spin partners, respectively.

#### 3.1 Sum rule for spin partner states

Taking the nodeless neutron orbits  $p$ ,  $d$  and  $f$  of  $^{48}\text{Ca}$  as examples, Table 1 shows the tensor interaction matrix elements  $V_{j_{\pm}j'}^T$  (in units of  $10^{-1}$  MeV), described by the supplemented relativistic formalism for the nuclear tensor force components in the Fock diagram of the  $\sigma$ - $S$  coupling. The 2<sup>nd</sup> to 7<sup>th</sup> rows show the results calculated with the full nucleon spinors. It is found that there exist rather distinct deviations from the sum rule, with the  $V_{\text{sum}}$  values comparable to the interaction matrix elements themselves. If one neglects the lower components in the nucleon spinors, i.e., the results in the 8<sup>th</sup> to 13<sup>th</sup>

rows, it can be seen that the sum rule is properly fulfilled, with relative deviations of a few percent. If we take the same assumption as in Ref. [60], i.e., that the  $j_+$  and  $j_-$  orbits, as well as the  $j'_+$  and  $j'_-$  orbits, share the same radial wave functions, in addition to neglecting the lower components, the sum rule is reproduced precisely with negligible errors ( $V_{\text{sum}} \lesssim 10^{-6}$  MeV).

Table 1. Interaction matrix elements  $V_{j_{\pm}j'}^T$  ( $10^{-1}$  MeV) between the spin partner states, namely the nodeless neutron orbits  $p$ ,  $d$  and  $f$  of  $^{48}\text{Ca}$ , for the tensor force components in the Fock diagram of the  $\sigma$ - $S$  couplings. The 2<sup>nd</sup>-7<sup>th</sup> rows show the results calculated with the radial wave functions determined by the self-consistent calculations of DDRHF with PKA1. For the results in the 8<sup>th</sup>-13<sup>th</sup> rows, the lower components in both the  $j_{\pm}$  and the  $j'_{\pm}$  orbits are omitted, and for those in the 14<sup>th</sup>-19<sup>th</sup> rows the  $j_{\pm}$  orbits, as well as  $j'_{\pm}$  orbits, share the same radial wave functions in addition to neglecting the lower components.

$j_{\pm}$	$j'$				
	$\nu 1p_{3/2}$	$\nu 1p_{1/2}$	$\nu 1d_{5/2}$	$\nu 1d_{3/2}$	$\nu 1f_{7/2}$
$\nu 1p_{3/2}$	0.463	-1.391	0.401	-1.031	0.259
$\nu 1p_{1/2}$	-1.391	2.000	-1.310	1.113	-0.884
$V_{\text{sum}}$	-0.929	-1.563	-1.015	-1.900	-0.731
$\nu 1d_{5/2}$	0.401	-1.310	0.691	-1.365	0.597
$\nu 1d_{3/2}$	-1.031	1.113	-1.365	1.261	-1.189
$V_{\text{sum}}$	-1.719	-3.409	-1.312	-3.146	-1.175
$\nu 1p_{3/2}$	0.635	-1.335	0.622	-0.963	0.433
$\nu 1p_{1/2}$	-1.335	2.818	-1.263	1.968	-0.848
$V_{\text{sum}}$	-0.132	0.298	-0.038	0.087	0.036
$\nu 1d_{5/2}$	0.622	-1.263	0.875	-1.295	0.795
$\nu 1d_{3/2}$	-0.963	1.968	-1.295	1.932	-1.142
$V_{\text{sum}}$	-0.118	0.295	0.072	-0.041	0.204
$\nu 1p_{3/2}$	0.635	-1.269	0.622	-0.933	0.433
$\nu 1p_{1/2}$	-1.269	2.538	-1.244	1.866	-0.866
$V_{\text{sum}}$	-0.000	0.000	0.000	0.000	0.000
$\nu 1d_{5/2}$	0.622	-1.244	0.875	-1.313	0.795
$\nu 1d_{3/2}$	-0.933	1.866	-1.313	1.969	-1.193
$V_{\text{sum}}$	0.000	0.000	-0.000	-0.000	0.000

Similar to Table 1, Table 2 shows the tensor interaction matrix elements described by the relativistic formalism for the nuclear tensor force components in the Fock diagram of the  $\omega$ - $V$  coupling, and similar systematics in reproducing the sum rule are found. In fact, besides the sum rule in Eq. (12), the interaction matrix elements of the tensor force should fulfill the following relations as well,

$$\hat{j}_+^2 \hat{j}_+^{\prime 2} V_{j_+j'_+}^T - \hat{j}_-^2 \hat{j}_-^{\prime 2} V_{j_-j'_-}^T = 0, \quad \hat{j}_+^2 \hat{j}_-^{\prime 2} V_{j_+j'_-}^T - \hat{j}_-^2 \hat{j}_+^{\prime 2} V_{j_-j'_+}^T = 0, \quad (13)$$

which are precisely reproduced by the proposed relativistic formalism, if both  $j_{\pm}$  and  $j'_{\pm}$  states share the same radial wave functions in addition to neglecting the lower

components (see the last six rows in Tables 1 and 2). Moreover, comparing the results of  $V_{jj'}^T$  with full nucleon spinors and those neglecting the lower components in Tables 1 and 2, it is seen that the influence induced by the inclusion of the lower components is more remarkable for the repulsive-type tensor interaction matrix elements, i.e.,  $V_{j_+j_+}^T$  and  $V_{j_-j_-}^T$  in Table 1, and  $V_{j_-j_+}^T$  and  $V_{j_+j_-}^T$  in Table 2, than for the attractive-type ones.

Table 2. Interaction matrix elements  $V_{j_{\pm}j'}^T$  ( $10^{-1}$  MeV) between the spin partner states, namely the nodeless neutron orbits  $p$ ,  $d$  and  $f$  of  $^{48}\text{Ca}$ , for the tensor force components in the Fock diagram of the  $\omega$ - $V$  couplings. The 2<sup>nd</sup>-7<sup>th</sup> rows show the results calculated with the radial wave functions determined by the self-consistent calculations of DDRHF with PKA1. For the results in the 8<sup>th</sup>-13<sup>th</sup> rows, the lower components in both the  $j_{\pm}$  and the  $j'_{\pm}$  orbits are omitted, and for those in the 14<sup>th</sup>-19<sup>th</sup> rows the  $j_{\pm}$  orbits, as well as  $j'_{\pm}$  orbits, share the same radial wave functions in addition to neglecting the lower components.

$j_{\pm}$	$j'$				
	$\nu 1p_{3/2}$	$\nu 1p_{1/2}$	$\nu 1d_{5/2}$	$\nu 1d_{3/2}$	$\nu 1f_{7/2}$
$\nu 1p_{3/2}$	-0.247	0.353	-0.250	0.150	-0.181
$\nu 1p_{1/2}$	0.353	-1.278	0.451	-0.935	0.341
$V_{\text{sum}}$	-0.282	-1.144	-0.101	-1.272	-0.042
$\nu 1d_{5/2}$	-0.250	0.451	-0.361	0.389	-0.339
$\nu 1d_{3/2}$	0.150	-0.935	0.389	-0.904	0.410
$V_{\text{sum}}$	-0.904	-1.036	-0.613	-1.285	-0.392
$\nu 1p_{3/2}$	-0.256	0.540	-0.265	0.412	-0.194
$\nu 1p_{1/2}$	0.540	-1.145	0.540	-0.845	0.381
$V_{\text{sum}}$	0.057	-0.128	0.020	-0.044	-0.014
$\nu 1d_{5/2}$	-0.265	0.540	-0.380	0.564	-0.357
$\nu 1d_{3/2}$	0.412	-0.845	0.564	-0.844	0.513
$V_{\text{sum}}$	0.058	-0.144	-0.026	0.008	-0.092
$\nu 1p_{3/2}$	-0.256	0.512	-0.265	0.397	-0.194
$\nu 1p_{1/2}$	0.512	-1.024	0.530	-0.794	0.388
$V_{\text{sum}}$	-0.000	0.000	-0.000	0.000	0.000
$\nu 1d_{5/2}$	-0.265	0.530	-0.380	0.570	-0.357
$\nu 1d_{3/2}$	0.397	-0.794	0.570	-0.855	0.536
$V_{\text{sum}}$	-0.000	-0.000	0.000	0.000	0.000

In order to understand the distinct difference of the sum rule (Eq. (12)) with/without the lower components of nucleon spinors in Tables 1 and 2, Fig. 2 shows the radial wave functions for the upper and lower components. The upper radial wave functions  $G(r)$  of the spin partner states are nearly identical to each other, which explains the correct reproduction of the sum rule shown in the middle six rows of Tables 1 and 2. However, the radial wave functions  $F(r)$  of the lower components, which emerge naturally with the relativistic treatment of the nucleon field, are quite different for the spin partners. This difference in  $F(r)$  is ascribed mainly to the so-called nodal effect, that is, the node numbers of  $G(r)$

and  $F(r)$  are the same for  $j_+$  states but different by one for  $j_-$  states. Therefore, although the component  $V_{G_j G_{j'}}^T$  in Eq. (10) fulfills appropriately the tensor sum rule in Eq. (12), those  $V_{jj'}^T$  components involving  $F(r)$ , namely  $V_{G_j F_{j'}}^T$ ,  $V_{F_j G_{j'}}^T$  and  $V_{F_j F_{j'}}^T$ , could break the sum rule impressively.

Despite the remarkable violation of the sum rule with full nucleon spinors, it is still found that the sum rule is in general better fulfilled for the couplings  $V_{j_{\pm}j'_+}^T$  than for  $V_{j_{\pm}j'_-}^T$ . This can be understood directly by looking through the role of  $V_{G_j F_{j'}}^T$ , since the contributions from the other components in Eq. (10) are similar in both cases. As has been claimed by the nodal effect, the structure of the lower radial wave functions  $F_{j'_-}(r)$  in  $j'_-$  states is actually more complicated than  $F_{j'_+}(r)$  in  $j'_+$  states, consequently leading to the sum rule being violated further for  $V_{G_{j_{\pm}} F_{j'_-}}^T$  than  $V_{G_{j_{\pm}} F_{j'_+}}^T$ .

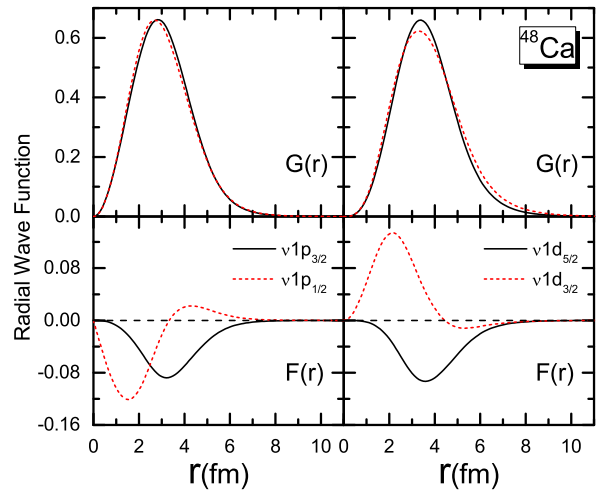


Fig. 2. (color online) Radial wave functions of the spin partner states  $\nu 1p$  and  $\nu 1d$  in  $^{48}\text{Ca}$ , where  $G(r)$  denotes the upper components and  $F(r)$  corresponds to the lower components. The results are extracted from the calculations of DDRHF with PKA1.

The sum rule is also checked for the case where the full Dirac spinors are used but assuming that the  $j_{\pm}$  orbits share the same radial wave functions, i.e., the lower components of the spin partner states are set to be the same rather than omitted, which actually can be regarded as eliminating the nodal effect of Dirac spinors. It is found that there still exists distinct violation of the sum rule shown in Eq. (12). Hence, the result implies the tensor interaction matrix elements from the lower components of Dirac spinors might correlate with a new tensor sum rule, which could be different from the case of the upper components of Dirac spinors and deserves to be investigated further.

### 3.2 Sum rule for pseudo-spin partner states

In nuclear single-particle spectra, the near degeneracy between the single-particle states with quantum numbers  $(n, l, j=l+1/2)$  and  $(n-1, l+2, j=l+3/2)$  has been recognized as pseudo-spin symmetry (PSS), and the doublet states are referred as pseudo-spin (PS) partners with newly defined quantum numbers  $\tilde{n}=n-1$ ,  $\tilde{l}=l+1$ ,  $\tilde{j}_{\pm}=\tilde{l}\pm 1/2$  [61, 62]. Nowadays, PSS is widely accepted as a relativistic symmetry [63], and the quantum number  $\tilde{l}$  is nothing but the orbit angular momentum of the lower components of the Dirac spinor. In recent decades, lots of efforts have been devoted to understanding the nature of PSS and the conservation conditions within the relativistic mean field models with/without the Fock terms [9, 34, 43, 44, 49, 64–75]. Similar to the spin partners, it is also worth checking the sum rule for the pseudo-spin partner states with the proposed relativistic formalism of the tensor forces, which could be helpful to understand the role of the tensor force in determining the PSS. Correspondingly, the sum rule for pseudo-spin partners can be expressed as,

$$\tilde{V}_{\text{sum}} \equiv \hat{j}_+^2 V_{\tilde{j}_+ \tilde{j}'}^T + \hat{j}_-^2 V_{\tilde{j}_- \tilde{j}'}^T = 0, \quad (14)$$

where  $\hat{j}^2 = 2\tilde{j}+1$ , and  $\tilde{j}_{\pm}$  denotes the pseudo-spin partners.

Taking the neutron pseudo-spin doublets  $1\tilde{p}$  and  $1\tilde{d}$  in  $^{90}\text{Zr}$  as examples, Table 3 shows the interaction matrix elements  $V_{\tilde{j}_{\pm} \tilde{j}'}^T$  between the pseudo-spin partners, calculated with the supplemented relativistic formalism for the tensor force components in the Fock diagram of the  $\sigma$ - $S$  coupling. In Table 3, the 2<sup>nd</sup> to 7<sup>th</sup> rows show the results (in units of  $10^{-1}$  MeV) calculated with the full nucleon spinors. It is found that the sum rule is violated completely, with the  $\tilde{V}_{\text{sum}}$  values one order of magnitude larger than the  $V_{\tilde{j}_{\pm} \tilde{j}'}^T$  values, and even the spin-dependent characteristic of the tensor forces is not regular, as in the cases of spin partners, any more. However, if one neglects the upper components of nucleon spinors, which correspond to the results (in units of  $10^{-4}$  MeV) in the 8<sup>th</sup> to 13<sup>th</sup> rows, the agreement with the sum rule is improved remarkably, while relatively less good than the cases neglecting the lower components of nucleon spinors for spin partners (see the 8<sup>th</sup>–13<sup>th</sup> rows in Tables 1 and 2). Furthermore, if the  $\tilde{j}_+$  and  $\tilde{j}_-$  orbits, as well as the  $\tilde{j}'_+$  and  $\tilde{j}'_-$  orbits, share the same radial wave functions, i.e., the results (in units of  $10^{-4}$  MeV) in the 14<sup>th</sup>–19<sup>th</sup> rows in Table 3, eventually the sum rule is reproduced precisely with negligible errors ( $\tilde{V}_{\text{sum}} \lesssim 10^{-9}$  MeV). As depicted in Table 3, similar systematics in describing the sum rule, although the results are not shown here, are found with the relativistic formalism for the tensor force components in the Fock diagrams of  $\omega$ - $V$ ,  $\rho$ - $V$ ,  $\rho$ - $T$  and  $\pi$ - $PV$  couplings. It is also found, in the last six rows of Table 3, that the relations (13) for pseudo-spin partners are

fulfilled precisely if one neglects the upper components and takes the same radial wave functions for the lower components.

Table 3. Interaction matrix elements  $V_{\tilde{j}_{\pm} \tilde{j}'}^T$  between the pseudo-spin partner states, namely the pseudo orbital  $\tilde{p}$  and  $\tilde{d}$  of  $^{90}\text{Zr}$ , for the tensor force components in the Fock diagram of the  $\sigma$ - $S$  couplings. The 2<sup>nd</sup>–7<sup>th</sup> rows show the results (in units of  $10^{-1}$  MeV) calculated with the radial wave functions determined by the self-consistent calculations of DDRHF with PKA1. For the results in the 8<sup>th</sup>–13<sup>th</sup> rows the upper components in both the  $\tilde{j}_{\pm}$  and the  $\tilde{j}'_{\pm}$  orbits are omitted, and for those in the 14<sup>th</sup>–19<sup>th</sup> rows the  $\tilde{j}_{\pm}$  orbits, as well as  $\tilde{j}'_{\pm}$  orbits, share the same radial wave functions in addition to neglecting the upper components; both are in units of  $10^{-4}$  MeV.

$\tilde{j}_{\pm}$	$\tilde{j}'$			
	$\nu 1\tilde{p}_{3/2}$	$\nu 1\tilde{p}_{1/2}$	$\nu 1\tilde{d}_{5/2}$	$\nu 1\tilde{d}_{3/2}$
$\nu 1\tilde{p}_{3/2}$	0.929	-0.142	0.672	-0.400
$\nu 1\tilde{p}_{1/2}$	-0.142	-0.460	-0.128	-0.396
$\tilde{V}_{\text{sum}}$	3.433	-1.487	2.432	-2.390
$\nu 1\tilde{d}_{5/2}$	0.672	-0.128	0.730	-0.272
$\nu 1\tilde{d}_{3/2}$	-0.400	-0.396	-0.272	0.020
$\tilde{V}_{\text{sum}}$	2.432	-2.351	3.290	-1.555
$\nu 1\tilde{p}_{3/2}$	0.931	-1.513	0.970	-1.161
$\nu 1\tilde{p}_{1/2}$	-1.513	2.472	-1.578	1.902
$\tilde{V}_{\text{sum}}$	0.698	-1.106	0.722	-0.842
$\nu 1\tilde{d}_{5/2}$	0.970	-1.578	1.591	-1.872
$\nu 1\tilde{d}_{3/2}$	-1.161	1.902	-1.872	2.229
$\tilde{V}_{\text{sum}}$	1.172	-1.863	2.057	-2.315
$\nu 1\tilde{p}_{3/2}$	0.618	-1.236	0.634	-0.951
$\nu 1\tilde{p}_{1/2}$	-1.236	2.472	-1.268	1.902
$\tilde{V}_{\text{sum}}$	-0.000	0.000	-0.000	-0.000
$\nu 1\tilde{d}_{5/2}$	0.634	-1.268	0.991	-1.486
$\nu 1\tilde{d}_{3/2}$	-0.951	1.902	-1.486	2.229
$\tilde{V}_{\text{sum}}$	0.000	-0.000	0.000	0.000

In order to understand the systematics of describing the sum rule for pseudo-spin partners, Fig. 3 shows the radial wave functions  $G(r)$  and  $F(r)$  as functions of radial distance  $r$ . The radial wave functions  $G(r)$  for the upper components are quite different for the pseudo-spin partners, from which the distinct violation of the sum rule can be well understood. However, as shown in the lower panels of Fig. 3, the lower components of the pseudo-spin partners are of similar radial dependence, which accounts for the approximate PSS. It is also interesting that the radial wave functions  $F(r)$  for the pseudo-spin partners are quantitatively different at large radial distance. As a result, the sum rule for the pseudo-spin partners, if neglecting the upper components, is not fulfilled as well as for the spin partners neglecting the lower components. Since the radial wave functions  $F(r)$  are nearly one order of magnitude smaller than the upper ones  $G(r)$ , it is not surprising that the interaction matrix elements  $V_{\tilde{j}_{\pm} \tilde{j}'}^T$  are

reduced by 3 orders of magnitude, if the upper components of nucleon spinors are neglected.

Within the non-relativistic scheme [2] the sum rule (12), as well as the relations (13), can be properly fulfilled, particularly if one adopts the same radial wave functions for the spin partners. For the pseudo-spin partners, however, as depicted in the 2<sup>nd</sup>–7<sup>th</sup> rows of Table 3, the relations (12) and (13) are not satisfied any more, due to the nodal difference. One may argue that with identical wave functions these relations can be fulfilled. However, such an arbitrary approximation already changes the pseudo-spin doublet states themselves. In the relativistic scheme, the relations (12) and (13) can be fulfilled properly for the spin partners if the lower components of the nucleon spinors are neglected, and can be reproduced precisely if one further lets the spin partners share the same radial wave functions  $G(r)$ . Moreover, for the pseudo-spin partners, the sum rules can be also fulfilled to certain extent if one neglects the upper components, and further these relations can be satisfied precisely with identical radial wave functions  $F(r)$  that correspond to the exact PSS.

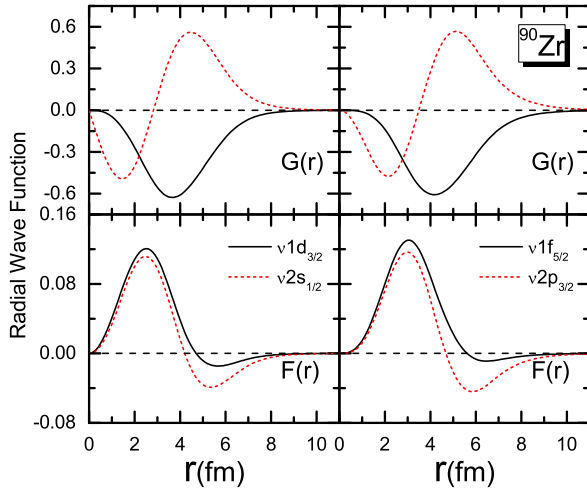


Fig. 3. (color online) Radial wave functions of the pseudo-spin partner states  $\nu 1\tilde{p}$  and  $\nu 1\tilde{d}$  in  $^{90}\text{Zr}$ , where  $G(r)$  denotes the upper components and  $F(r)$  corresponds to the lower components. The results are extracted from the calculations of DDRHF with PKA1.

### 3.3 Sum rule for the nodal states

As discussed in the previous subsections, the nodal differences in the lower components of spin-partners, as well as in the upper components of the pseudo-spin partners, induce distinct violations of the sum rules (12) and (13). It is then worth checking the validity of the sum rules for the nodal states with the relativistic formalism for the tensor force components in the Fock diagrams.

Taking the nodeless neutron orbit  $\nu 1h$ , and the nodal orbits  $\nu 2d$  and  $\nu 3p$  in  $^{208}\text{Pb}$  as examples, Tables 4–6 shows the interaction matrix elements  $V_{j_{\pm}j'}^T$  (in units of  $10^{-2}$  MeV), calculated with the supplemented relativistic formalism for the tensor force components in the Fock diagram of the  $\sigma$ - $S$  coupling. As expected, the magnitude of  $V_{j_{\pm}j'}^T$  tends to be smaller with increasing mass numbers, i.e., the units changing from  $10^{-1}$  MeV to  $10^{-2}$  MeV. Similar to the results in the 2<sup>nd</sup> to 7<sup>th</sup> rows in Table 1, Table 4 shows the results calculated with the full nucleon spinors, and distinct deviations from the sum rule can be seen with the  $V_{\text{sum}}$  values comparable to  $V_{j_{\pm}j'}^T$  themselves. Notice that the sum rules are scaled with the occupations  $\hat{j}_{\pm}^2$ , see Eqs. (12) and (13), and as a result the violation of the sum rules then increases with increasing angular momentum, particularly for the high- $j$  orbits  $\nu 1h$  and  $\nu 2f$ . For  $V_{j_{\pm}j'}^T$  between the  $\nu 3p$  orbits, the deviations from the sum rule are also surprisingly large.

Table 4. Interaction matrix elements  $V_{j_{\pm}j'}^T$  ( $10^{-2}$  MeV) between the spin partner states, namely the nodeless neutron orbit  $1h$  and the nodal neutron orbits  $2d$  and  $3p$  of  $^{208}\text{Pb}$ , for the tensor force components in the Fock diagram of the  $\sigma$ - $S$  couplings. The results are calculated with the radial wave functions determined by the self-consistent calculations of DDRHF with PKA1.

$j_{\pm}$	$j'$					
	$\nu 1h_{11/2}$	$\nu 1h_{9/2}$	$\nu 2f_{7/2}$	$\nu 2f_{5/2}$	$\nu 3p_{3/2}$	$\nu 3p_{1/2}$
$\nu 1h_{11/2}$	3.048	-4.884	0.617	-1.603	-0.089	-0.713
$\nu 1h_{9/2}$	-4.884	3.673	-1.971	0.918	-0.778	0.021
$V_{\text{sum}}$	-12.26	-21.88	-12.31	-10.05	-8.850	-8.354
$\nu 2f_{7/2}$	0.617	-1.971	1.114	-3.344	-0.231	-1.298
$\nu 2f_{5/2}$	-1.603	0.918	-3.344	2.186	-1.237	0.187
$V_{\text{sum}}$	-4.680	-10.26	-11.15	-13.63	-9.274	-9.266
$\nu 3p_{3/2}$	-0.089	-0.778	-0.231	-1.237	-0.007	-4.389
$\nu 3p_{1/2}$	-0.713	0.021	-1.298	0.187	-4.389	3.600
$V_{\text{sum}}$	-1.782	-3.072	-3.521	-4.577	-8.806	-10.35

Furthermore, if we neglect the lower components of the nucleon spinors, i.e., the results in Table 5, the agreements with the sum rule are distinctly improved, despite the large scaling factors in high- $j$  orbits. Specifically, for  $V_{j_{\pm}j'}^T$  between the  $\nu 3p$  orbits, the deviations from the sum rule tend to vanish. In fact it can also be found that for the repulsive-type  $V_{j_{\pm}j'}^T$ , namely  $V_{j_{+}j'_{+}}^T$  and  $V_{j_{-}j'_{-}}^T$ , the contributions from the lower component are also remarkable, and even change the sign of the interaction matrix elements (see Table 4). The  $V_{\text{sum}}$  values for the high- $j$  orbits  $\nu 1h$  and  $\nu 2f$  are in general larger than those for  $\nu 3p$  orbits, see Table 5. This can be clearly understood from the radial wave functions  $G(r)$  shown in Fig. 4. It is shown that the differences in the wave functions  $G(r)$  of the spin partners  $\nu 1h$  are large, due to the fact that the  $\nu 1h$  orbits cross over the major shell  $N=82$ , whereas the

$\nu 3p$  orbits have almost identical  $G(r)$ . If one lets the  $j_+$  and  $j_-$  orbits, as well as  $j'_+$  and  $j'_-$  orbits, share the same radial wave functions in addition to neglecting the lower components, as shown in Table 6, the sum rules can be reproduced precisely with negligible errors ( $V_{\text{sum}} \lesssim 10^{-7}$  MeV).

Table 5. Interaction matrix elements  $V_{j_{\pm}j'}^T$  ( $10^{-2}$  MeV) between the spin partner states, namely the nodeless neutron orbit  $1h$  and the nodal neutron orbits  $2d$  and  $3p$  of  $^{208}\text{Pb}$ , for the tensor force components in the Fock diagram of the  $\sigma$ - $S$  couplings, with the lower components of nucleon spinors in both  $j_{\pm}$  and  $j'_{\pm}$  orbits omitted. The results are calculated with the radial wave functions determined by the self-consistent calculations of DDRHF with PKA1.

$j_{\pm}$	$j'$					
	$\nu 1h_{11/2}$	$\nu 1h_{9/2}$	$\nu 2f_{7/2}$	$\nu 2f_{5/2}$	$\nu 3p_{3/2}$	$\nu 3p_{1/2}$
$\nu 1h_{11/2}$	3.820	-4.553	1.086	-1.384	0.260	-0.503
$\nu 1h_{9/2}$	-4.553	5.516	-1.453	1.807	-0.339	0.655
$V_{\text{sum}}$	0.317	0.529	-1.504	1.463	-0.268	0.509
$\nu 2f_{7/2}$	1.086	-1.453	2.263	-2.880	0.462	-0.889
$\nu 2f_{5/2}$	-1.384	1.807	-2.880	3.703	-0.611	1.173
$V_{\text{sum}}$	0.383	-0.786	0.828	-0.817	0.033	-0.072
$\nu 3p_{3/2}$	0.260	-0.339	0.462	-0.611	1.818	-3.533
$\nu 3p_{1/2}$	-0.503	0.655	-0.889	1.173	-3.533	6.872
$V_{\text{sum}}$	0.033	-0.045	0.070	-0.095	0.204	-0.389

Table 6. Interaction matrix elements  $V_{j_{\pm}j'}^T$  ( $10^{-2}$  MeV) between the spin partner states, namely the nodeless neutron orbit  $1h$  and the nodal neutron orbits  $2d$  and  $3p$  of  $^{208}\text{Pb}$ , for the tensor force components in the Fock diagram of the  $\sigma$ - $S$  couplings, with the lower components of nucleon spinors in both  $j_{\pm}$  and  $j'_{\pm}$  orbits omitted, and the  $j_{\pm}$  orbits and  $j'_{\pm}$  orbits sharing the same radial wave functions. The results are calculated with the radial wave functions determined by the self-consistent calculations of DDRHF with PKA1.

$j_{\pm}$	$j'$					
	$\nu 1h_{11/2}$	$\nu 1h_{9/2}$	$\nu 2f_{7/2}$	$\nu 2f_{5/2}$	$\nu 3p_{3/2}$	$\nu 3p_{1/2}$
$\nu 1h_{11/2}$	3.820	-4.584	1.086	-1.448	0.260	-0.520
$\nu 1h_{9/2}$	-4.584	5.501	-1.303	1.737	-0.312	0.624
$V_{\text{sum}}$	0.000	0.000	0.000	-0.000	-0.000	-0.000
$\nu 2f_{7/2}$	1.086	-1.303	2.263	-3.018	0.462	-0.924
$\nu 2f_{5/2}$	-1.448	1.737	-3.018	4.024	-0.616	1.232
$V_{\text{sum}}$	0.000	0.000	0.000	0.000	-0.000	-0.000
$\nu 3p_{3/2}$	0.260	-0.312	0.462	-0.616	1.818	-3.636
$\nu 3p_{1/2}$	-0.520	0.624	-0.924	1.232	-3.636	7.271
$V_{\text{sum}}$	-0.000	0.000	-0.000	-0.000	0.000	0.000

Similar tests were also performed with the relativistic formalism for the tensor force components in the Fock diagrams of the  $\omega$ - $V$ ,  $\rho$ - $V$ ,  $\pi$ - $PV$  and  $\rho$ - $T$  couplings in DDRHF with PKA1, and similar systematics are found

for the nodal orbits on the tensor sum rule. Following the characteristics of tensor force, the interactions between the partners  $\{j_{\pm}, j'_{\pm}\}$  are opposite to the ones between the partners  $\{j_{\pm}, j'_{\mp}\}$ . Notice that the states  $j_{\pm}$  are often denoted with the quantum number  $\kappa_{\pm}$ , with  $\kappa_{\pm} = \mp(j+1/2)$ . In addition, with the relativistic representation of nucleon spinors, the signs of  $\kappa$  are opposite for the upper and lower components. Thus, according to the spin-dependent feature of the tensor force, the signs of  $V_{G_j G_{j'}}^T$  and  $V_{F_j F_{j'}}^T$  are different from the signs of  $V_{G_j F_{j'}}^T$  and  $V_{F_j G_{j'}}^T$ . So the dialog between the upper component of the  $j$  ( $j'$ ) state and the lower component of the  $j'$  ( $j$ ) state, namely  $V_{G_j F_{j'}}^T$  ( $V_{F_j G_{j'}}^T$ ), will partly cancel the dominant contributions of  $V_{G_j G_{j'}}^T$  between the upper components of states  $j$  and  $j'$  via the tensor force. As a result, distinct violations of the sum rules are then found in Tables 1-6 due to the interference between the upper and lower components of nucleon spinors.

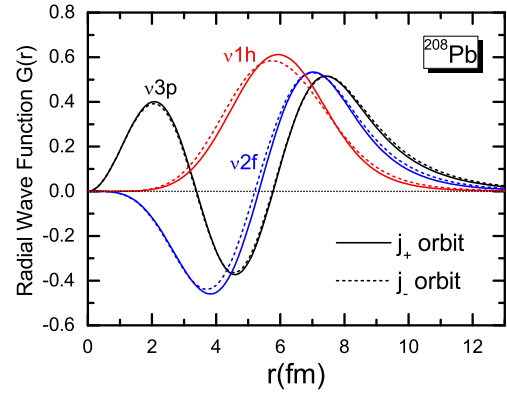


Fig. 4. (color online) Radial wave functions  $G(r)$  of the spin partner states  $\nu 1h$ ,  $\nu 2f$  and  $\nu 3p$  in  $^{208}\text{Pb}$ . The results are extracted from the calculations of DDRHF with PKA1.

## 4 Summary

In conclusion, the nature of nuclear tensor force has been illustrated in detail within the relativistic Hartree-Fock approach, with a series of relativistic formalisms of the tensor forces which are naturally introduced by the Fock diagrams of meson-nucleon couplings. Taking the original wave functions determined by the self-consistent DDRHF calculations, namely without dropping the lower components of Dirac spinors, the contributions to the spin-orbit splitting from the Fock diagrams have been analyzed in selected realistic nuclei, and it has been shown that the spin-dependent feature described by the relativistic formalism for the tensor force is overestimated in the  $\sigma$ - $S$  coupling channel. Drawing inspiration from different ranks of Lorentz tensor couplings used in



various meson coupling channels, the relativistic formalism for the tensor force component in the  $\sigma$ - $S$  channel was then supplemented to its higher rank, and proved to be impressive in fully interpreting the spin-dependent feature of the tensor force brought about by the Fock terms in realistic nuclei, without ignoring the lower components of nucleon spinors.

Taking the doubly magic nuclei  $^{48}\text{Ca}$  and  $^{208}\text{Pb}$  and the semi-magic one  $^{90}\text{Zr}$  as candidates, the tensor sum rules were then tested for the spin and pseudo-spin partners with and without nodes, to further investigate the tensor force nature within the relativistic model. Due to the opposite signs of the  $\kappa$  quantities of the upper and lower components, as well as the nodal difference, it is

shown that the interference between the two components of nucleon spinors brings distinct violations of the tensor sum rules in realistic nuclei. The spin dependence in the contributions to the spin-orbit splittings from the Fock diagrams can be almost fully taken into account by the supplemented relativistic formalism of the tensor force components. Moreover, if one neglects the lower/upper components of nucleon spinors for the spin/pseudo-spin partners, the sum rules can be fulfilled properly, and can be precisely reproduced if, in addition, the same radial wave functions are taken for the spin/pseudo-spin partners, clearly revealing the nature of the tensor force, and illustrating the validity of the relativistic formalism of nuclear tensor forces.

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