Hyperon Effects on the Spin Parameter of Rotating Neutron Stars *

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Based on the equations of state from the relativistic mean field theory without and with the inclusion of strangeness-bearing hyperons, we study the dimensionless spin parameter $j = cJ/(GM^2)$ of uniformly rotating neutron stars. It is shown that the maximum value of the spin parameter $j_{\rm max}$ of a neutron star rotating at the Keplerian frequency $f_{\rm K}$ is $j_{\rm max} \sim 0.7$ when the star mass $M > 0.5M_\odot$, which is sustained for various versions of equations of state without and with hyperons. The relationship between j and the scaled rotation frequency $f/f_{\rm K}$ is found to be insensitive to the star mass or the adopted equation of state in the models without hyperons. However, the emergence of hyperons in neutron stars will lead to an uncertainty of the spin parameter j, which in turn could generate a complexity in the theoretical study of the quasi-periodic oscillations observed in disk-accreting compact-star systems.

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Neutron stars, as one kind of the most exotic objects in the universe, play the role of a bridge between nuclear physics and astrophysics. In recent decades, many millisecond pulsars have been reported,[1−3] which in theoretical studies usually are served as compact relativistic stars with a strong magnetic field, fast rotation frequency, and strong gravitational binding. Various numerical codes have been developed recently to construct rapidly rotating neutron star models in general relativity (see Ref. [4] for a review).

As the rotation frequency f of pulsars is a directly measurable quantity, the maximum rotation frequency $f_{\rm K}$ (Keplerian frequency) for relativistic rotating neutron stars has attracted plentiful interest in previous studies.[5−7] In addition, another interesting physical quantity, that is, the dimensionless spin parameter $j = cJ/(GM^2)$, where J is the angular momentum and M is the gravitational mass of neutron stars, was introduced into the investigative territory of compact stars,[5,8] which plays an important role in understanding the observed quasi-periodic oscillations (QPOs) in disk-accreting compact-star systems.[9,10] In Ref. [8] it was revealed that the maximum value of the spin parameter of a traditional neutron star (composed by β-equilibrium nucleon matter) rotating at the Keplerian frequency is $j_{\rm max} \sim 0.7$, and the value is essentially independent of the mass of the neutron star as long as the mass is larger than about $1M_\odot$. However, the spin parameter of a quark star modeled by the MIT bag model does not have a universal upper bound and could be larger than unity. Thus a determination of the spin parameter from observed QPOs combined with an independent measure of the mass from other observables for rapidly rotating compact stars could provide strict constraints on the equation of state (EOS) of dense matter.[9,10] Recently, the authors of Ref. [11] also confirmed $j_{\rm max} \sim 0.7$ for a traditional neutron star.

During recent years, the relativistic many-body theory has achieved great success for the description of nuclear matter and finite nuclei. One of the most successful representatives is the relativistic Hartree approach with the no-sea approximation, i.e., the relativistic mean field (RMF) theory.[12−14] With a limited number of free parameters including the meson masses and meson-nucleon coupling constants, the appropriate quantitative descriptions are obtained by RMF in describing nuclear matter and neutron stars.[15−25] It is generally believed that hyperons would appear at roughly twice the normal nuclear matter density in neutron-star matter.[26] There have been several works to study the effects of the inclusion of hyperons on non-rotating neutron stars.[15,27−31] For the rapidly rotating neutron stars, it is thus interesting to introduce the strangeness degree of freedom into the EOS and to investigate the corresponding spin properties of stars.[32]

In this Letter, the equations of state without and with the inclusion of strangeness-bearing hyperons based on the RMF theory will be applied to study the properties of rotating neutron stars, wherein close attention will be paid to the dimensionless spin pa-

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rameter j.

We compute numerically the rotating neutron stars with the RNS code (see Refs. [4,5,33,34] and references therein). The code solves the hydrostatic and Einstein's field equations for mass distributions rotating rigidly under the assumption of stationary and axial symmetry about the rotational axis, and reflection symmetry about the equatorial plane. Then the angular momentum J of a rotating neutron star can be calculated in the code as follows:

$$J = \int T^{\mu}_{\nu} \xi^{\nu}_{(\phi)} dV, \quad (1)$$

where T^{μ}_{ν} is the energy-momentum tensor of stellar matter, $\xi^{\nu}_{(\phi)}$ is the Killing vector in the azimuthal direction reflecting axial symmetry, and dV is a proper 3-volume element.

In this work, we employ six representative EOSs from the RMF theory to model rotating neutron stars, i.e., TW99,[35] PKDD,[36] and DD-ME2[37] from the RMF effective interactions with density-dependent meson-nucleon couplings, and GL97,[15] TM1[26] and PK1[36] from the RMF effective interactions with nonlinear self-couplings of the meson fields. In the calculations, the hyperons Λ, $\Sigma^{\pm}$, Σ^0, Ξ^- and Ξ^0 from the baryon octet are included. Following the previous study,[17] we introduce the ratios of the meson-hyperons coupling constants to those of nucleons, and the ratios $x_{\sigma h} = x_{\omega h} = x_{\rho h} = \sqrt{2/3}$ are chosen according to Ref. [38]. The detailed discussion for the application of these RMF effective interactions in neutron star physics can be found in Refs. [17,20].

Table 1. The maximum allowable gravitational mass $M_{\rm max}$ for rotating neutron stars, and the Keplerian frequency $f_{\rm K,max}$ ($f_{{\rm K},1.4M_\odot}$) for rotating neutron stars with the maximum ($1.4M_\odot$) gravitational mass obtained from different EOS models. For comparison, the results for neutron stars without and with hyperons are listed, respectively.

EOS	Without hyperon			With hyperon		
	$M_{\rm max}$ ($M_\odot$)	$f_{\rm K,max}$ (kHz)	$f_{{\rm K},1.4M_\odot}$ (kHz)	$M_{\rm max}$ ($M_\odot$)	$f_{\rm K,max}$ (kHz)	$f_{{\rm K},1.4M_\odot}$ (kHz)
TW99	2.48	1.67	0.93	2.20	1.43	0.83
PKDD	2.78	1.51	0.80	2.34	1.40	0.78
DD-ME2	3.02	1.51	0.86	2.53	1.40	0.81
GL97	2.37	1.55	0.82	1.87	1.37	0.82
TM1	2.60	1.32	0.74	1.78	1.13	0.74
PK1	2.78	1.34	0.74	1.88	1.15	0.75

We first show the influence of the inclusion of hyperons on the bulk properties of rotating neutron stars in Table 1. The maximum allowable gravitational masses $M_{\rm max}$ for rotating neutron stars range from $2.37M_\odot$ to $3.02M_\odot$ for the EOSs without hyperons, and the corresponding Keplerian frequencies $f_{\rm K,max}$ are located between 1.32 kHz and 1.67 kHz. Here by using EOSs with hyperons, the above regions move down to $M_{\rm max} \in [1.78, 2.53]M_\odot$ and $f_{\rm K,max} \in [1.13, 1.43]$ kHz, respectively. It is clearly seen that the results depend on the chosen EOSs, and due to the softening effects to the EOS, all of the values go down when hyperons are included in the calculations.

Now we turn to the dimensionless spin parameter $j = cJ/(GM^2)$. In Fig. 1, we plot the maximum angular momentum $J_{\rm max}$ of a neutron star rotating at the Keplerian frequency and the corresponding spin parameter $j_{\rm max}$ against the gravitational mass with various RMF EOSs without or with hyperons, where $J_{\rm max}$ is in units of $GM^2_\odot/c$. In Fig. 1, each line corresponds to one selected EOS and each point in the line corresponds to a star model with a fixed gravitational mass M rotating at its Keplerian frequency, while the black dot on the line denotes the maximum mass of a non-rotating neutron star.

From the left panel of Fig. 1, it is seen that $J_{\rm max}$ increases monotonically with increasing the neutron star mass, while $j_{\rm max}$ goes up quickly first when the neutron star mass $M < 0.5M_\odot$ and then stays stable in a narrow range $j_{\rm max} \sim 0.65$–0.72 after $M > 0.5M_\odot$ until the maximum allowable mass. At fixed neutron star mass, both $J_{\rm max}$ and $j_{\rm max}$ display almost the same results for six different EOS models.

In the right panel of Fig. 1, we plot the corresponding results for the neutron stars with hyperons. The spin properties $J_{\rm max}$ and $j_{\rm max}$ of neutron stars with hyperons seem to be very similar to those without hyperons except an imperceptible decrease at large gravitational mass. In both the cases without and with hyperons, the conclusion of $j_{\rm max} \sim 0.7$ for neutron stars suggested in Ref. [8] is supported very well in the present RMF calculations.

So far, we have studied $j_{\rm max}$ of neutron stars rotating at their Keplerian frequencies $f_{\rm K}$. In the following, the behaviors of spin parameter j for neutron stars with rotation frequency $f < f_{\rm K}$ will be investigated since realistic neutron stars usually rotate slower than their Keplerian frequencies. In Fig. 2, the spin parameter j is drawn as a function of the scaled rotation frequency $f/f_{\rm K}$ for different baryon mass of neutron stars, taking the results with DD-ME2 EOS as an example since the physics does not depend sensitively on EOSs. In Fig. 2, each line is labeled by the corresponding baryon mass M, that is, M=0.6, 1.0, 1.4, 1.8 and $2.2M_\odot$, respectively. Accordingly, $f_{\rm K}$ changes from ~ 0.56 kHz (for $0.6M_\odot$) to ~ 1.0 kHz (for $2.2M_\odot$)

for both the neutron stars without and with hyperons.

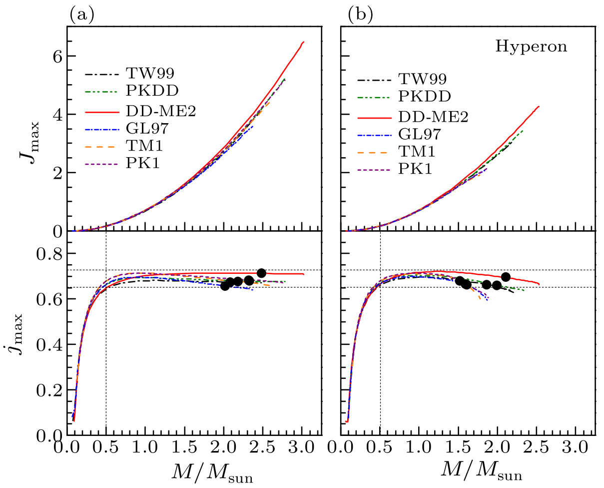

Fig. 1. The maximum values of the angular momentum $J_{\rm max}$ of a neutron star rotating at the Keplerian frequency and the corresponding spin parameter $j_{\rm max} = cJ_{\rm max}/(GM^2)$ as a function of the gravitational mass of rotating neutron stars. The results are given for various RMF EOSs without (left panel) or with hyperons (right panel).

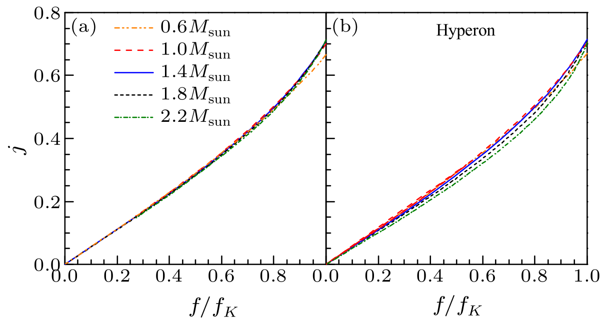

Fig. 2. Spin parameter as a function of the scaled rotation frequency for neutron stars constructed with DD-ME2 EOS. Each line represents a sequence of fixed baryon mass of the neutron star, that is, 0.6, 1.0, 1.4, 1.8 and $2.2M_\odot$.

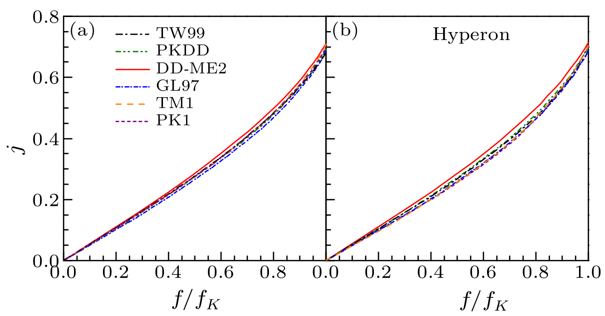

Fig. 3. Spin parameter as a function of the scaled rotation frequency for neutron stars constructed with six different EOSs. The baryon mass of the neutron star is fixed at $M = 1.4M_\odot$.

It is revealed in the left panel of Fig. 2 that the spin parameter changes slightly for different M configurations, except for the case of $0.6M_\odot$ with the scaled rotation frequency $f/f_{\rm K} \sim 1$, consistent with the previous studies.[8] With the inclusion of hyperons, the divergence of spin parameter j becomes larger as the star mass changes, as shown in the right panel. Thus for a fixed scaled rotation frequency $f/f_{\rm K}$, the emergence of hyperons in neutron stars breaks the universality and results in an uncertainty of the spin parameter so that more efforts should be carried out to the calculations of QPOs as the star mass or rotation frequency changes.

In Fig. 3, we plot the spin parameter j of neutron stars against $f/f_{\rm K}$ for six different EOS models, with the baryon mass fixed at $M = 1.4M_\odot$. The Keplerian frequency varies for different EOSs, changing from 0.71 to 0.89 kHz for both the neutron stars without and with hyperons. However, for a fixed rotation frequency, the spin parameter of neutron stars is essentially independent of the EOS models in the case without hyperons. With the inclusion of hyperons, the divergence of the spin parameter becomes slightly larger for the different EOSs.

In conclusion, the dimensionless spin parameter $j = cJ/(GM^2)$ of uniformly rotating neutron stars has been investigated based on various equations of state provided by the relativistic mean field theory without or with the inclusion of strangeness degree of freedom. It is shown that the maximum value of the spin parameter of a neutron star rotating at the Keplerian frequency is $j_{\rm max} \sim 0.7$, which is held for different versions of RMF EOS without and with hyperons, and is insensitive to the neutron star mass if $M > 0.5M_\odot$, in agreement with the previous study in Ref. [8] while with a wider scope of application for hypertonic neutron stars. The relationship between j and the scaled rotation frequency $f/f_{\rm K}$ is found to be insensitive to the star mass or the adopted equation of state in the models without hyperons. However, with the inclusion of hyperons, the spin parameter exhibits an uncertainty with respect to the star mass and the selected EOS. Thus such an error of j will introduce a complexity in the calculations of QPOs so that more efforts to the problem are required.

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