

## KEPLERIAN FREQUENCY OF UNIFORMLY ROTATING NEUTRON STARS IN RELATIVISTIC MEAN FIELD THEORY

N. B. ZHANG, B. QI\*, S. Y. WANG and S. L. GE

*School of Space Science and Physics, Shandong University,  
Weihai 264209, P. R. China*

\*bqi@sdu.edu.cn

B. Y. SUN

*School of Nuclear Science and Technology, Lanzhou University,  
Lanzhou 730000, P. R. China*

Received 4 September 2013

Accepted 6 November 2013

Published 20 November 2013

Adopting the equation of states (EOSs) from the relativistic mean field (RMF) theory, the relationships among the keplerian frequency  $f_K$ , gravitational mass  $M$  and radius  $R$  for the rapidly rotating neutron stars with and without hyperons are presented and analyzed. For various RMF EOSs, the empirical formula  $f_K(M) = 1.08 (M/M_\odot)^{1/2} (R_S/10 \text{ km})^{-3/2}$  kHz, proposed by P. Haensel *et al.* [*Astron. Astrophys.* **502** (2009) 605], is found to be an approximation with the error at most 13% and such approximation is worse for the neutron stars with hyperons. It indicates that the errors should be considered when the empirical formula is used to discuss the properties of neutron stars.

*Keywords:* Relativistic mean field theory; hyperons; neutron stars; keplerian frequency.

PACS Number(s): 21.30.Fe, 21.60.Jz, 26.60.-c, 97.60.Jd

### 1. Introduction

Neutron star remains one of the hot topics because they are the densest stellar objects in the universe. Due to their strong gravitational binding, neutron star can be very fast rotator. After the first millisecond pulsar B1937+21 with rotational frequency  $f = 641$  Hz was reported,<sup>1</sup> many millisecond pulsars have been observed.<sup>2</sup> So far, the fastest rotating compact star is XTE J1739-285 with  $f = 1122$  Hz reported in 2007.<sup>3</sup> Various numerical codes have been developed recently to construct rapidly rotating neutron star models in general relativity.<sup>4</sup>

As the rotation frequency  $f$  is a directly measurable quantity for pulsars, thus the maximum frequency, namely, the keplerian (mass-shedding) frequency  $f_K$ ,

has been one of the most studied physical quantities for rotating stars.<sup>5–10</sup> The relationships between  $f_K$  and the other measurable quantities, including gravitational mass  $M$  and radius  $R$ , are very interesting, which can be tested by the future observations. In theoretical calculations, the relationships of  $f_K \sim M$ ,  $M \sim R$ ,  $f_K \sim R$  all depend on the equation of states (EOSs). In 2004, Lattimer *et al.*<sup>11</sup> proposed an approximate empirical formula  $f_K(M) = f_0(M/M_\odot)^{1/2}(R_S/10 \text{ km})^{-3/2}$  with  $f_0 = 1.04 \text{ kHz}$ , which does not depend on the EOSs, where  $R_S$  is the radius of the nonrotating static star with mass  $M$ . Later, Haensel *et al.*<sup>9</sup> calculated precise two-dimensional models of rapidly rotating compact stars with 10 EOSs, and  $f_0 = 1.08 \text{ kHz}$  is suggested for the neutron stars. Such an empirical formula in which three measurable quantities are connected will be very useful if it is sufficiently precise and universal, thus it is worthwhile to systematically investigate based on more EOSs from the theories of dense matter.

On the side of the nuclear theory, the relativistic mean field (RMF) theory has achieved great success for the description of the nuclear matter and finite nuclei during the past years.<sup>12–28</sup> In the early development of RMF theory, it was applied to evaluate the total mass and radius of neutron stars.<sup>29</sup> In its further development, the RMF theory with density dependence in the meson–nucleon couplings (DDRMF)<sup>30–34</sup> and with the nonlinear self-coupling of  $\sigma$ ,  $\omega$  and  $\rho$  mesons (nonlinear RMF)<sup>22,35–38</sup> were developed and used to describe neutron stars, respectively. In addition, the consequences on compact star properties were studied with the inclusion of hyperons.<sup>39–42</sup> In Ref. 9, two RMF EOSs (GN3 and GNH3)<sup>40</sup> developed in 1985 have been used to study the above empirical formula.

In this paper, adopting various RMF EOSs with and without hyperons developed recently, we will present the relationships among  $f_K$ ,  $M$  and  $R$  for the neutron stars, and check the validity of the above interesting empirical formula. The paper is organized as follows: Section 2 introduces the EOSs of neutron stars from RMF theory used in our calculations. Section 3 provides the calculated  $f_K$ ,  $M$  and  $R$  of stationary rotating neutron stars with and without hyperons. In Sec. 4, we will check the validity of the above empirical formula against results from RMF EOSs, and discuss the correlative relationships. Finally, a summary is given in Sec. 5.

## 2. EOSs of Neutron Stars from RMF Theory

The RMF theory starts from an effective Lagrangian density with baryons, mesons ( $\sigma$ ,  $\omega$  and  $\rho$ ) and photons as degrees of freedom:

$$\begin{aligned} \mathcal{L} = & \sum_B \bar{\psi}_B \left[ i\gamma^\mu \partial_\mu - m_B - g_{\sigma B} \sigma - g_{\omega B} \gamma^\mu \omega_\mu - g_{\rho B} \gamma^\mu \tau_B \cdot \rho_\mu - e\gamma^\mu A_\mu \frac{1 - \tau_{3B}}{2} \right] \psi_B \\ & + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - U(\sigma) - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + U(\omega) \\ & - \frac{1}{4} \rho_{\mu\nu} \rho^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu - \frac{1}{4} A_{\mu\nu} A^{\mu\nu}, \end{aligned} \quad (1)$$

Table 1. The mass  $M_{\max}^{\text{stat}}$ , circumferential radius  $R_{\max}^{\text{stat}}$  of the nonrotating stars with maximum allowable masses and the radius  $R_{1.4}$  of nonrotating stars with  $1.4M_{\odot}$ .

Model	EOS	Refs.	Without hyperon			With hyperon		
			$M_{\odot}^{\text{stat}}$	$R_{\max}^{\text{stat}}$ (km)	$R_{1.4}$ (km)	$M_{\odot}^{\text{stat}}$	$R_{\max}^{\text{stat}}$ (km)	$R_{1.4}$ (km)
DDRMF	DDME2	24	2.49	12.05	13.27	2.09	11.68	13.17
	PKDD	22	2.33	11.78	13.72	1.97	11.48	13.52
	TW99	21	2.08	10.62	12.36	1.83	10.75	12.35
Nonlinear	GL97	29	2.02	11.02	13.29	1.61	10.48	12.77
RMF	PK1	22	2.32	12.63	14.44	1.59	12.10	13.45
	TM1	44	2.18	12.37	14.37	1.52	11.72	13.30

where the Dirac spinor  $\psi_B$  denotes the baryon  $B$  with mass  $m_B$  and isospin  $\tau_B$ . The sum on  $B$  is over protons, neutrons and hyperons ( $\Lambda$ ,  $\Sigma^{\pm}$ ,  $\Sigma^0$ ,  $\Xi^{-}$ ,  $\Xi^0$ , etc.) in this paper. For the description of neutron star matter, the coulomb field could be neglected. The details of RMF theory can be found in Refs. 19, 33 and 43.

In this paper, we employ three representative EOSs from the nonlinear RMF theory, GL97,<sup>29</sup> PK1<sup>22</sup> and TM1,<sup>44</sup> as well as three ones from the DDRMF theory, DDME2,<sup>24</sup> PKDD<sup>22</sup> and TW99.<sup>21</sup> The ratios of the meson–hyperons coupling constants to those of nucleons have been introduced,<sup>33</sup> and the ratios  $x_{\sigma h} = x_{\omega h} = x_{\rho h} = \sqrt{2/3}$  are chosen.<sup>45</sup> In the low-density region ( $\rho_b < 0.08 \text{ fm}^{-3}$ ), instead of RMF calculations, the Negele and Vautherin (NV) EOS for the inner crust<sup>46</sup> and Baym–Pethick–Sutherland (BPS) EOS<sup>47</sup> for the outer crust are chosen to join our EOSs. A neutron star is described as the  $\beta$ -stable nuclear matter system, which consists of not only baryons but also leptons  $\lambda$  (mainly  $e^{-}$  and  $\mu^{-}$ ). The calculations of energy density and pressure for neutron stars are shown in Refs. 33 and 43.

The properties for the EOSs from RMF theory have been discussed in detail, including the bulk properties, the density dependence of the coupling constant, the symmetry energy and the proton fraction,<sup>33,43</sup> which is very different for these six EOSs. Adopting these six EOSs, the calculated mass  $M_{\max}^{\text{stat}}$  and circumferential radius  $R_{\max}^{\text{stat}}$  of the nonrotating stars with maximum allowable masses and the radius  $R_{1.4}$  of nonrotating stars with  $1.4M_{\odot}$  are shown in Table 1, which demonstrate the different neutron star structures obtained from the different RMF EOSs.

### 3. Stationary Rotating Configurations of Neutron Stars

Numerical computations of the rotating neutron star based on the RMF EOSs have been performed using the RNS code, which is developed by Nikolaos Stergioulas and Sharon Morsink (<http://www.gravity.phys.uwm.edu/rns/>). In the code, stationary configurations of rigidly rotating neutron stars are computed in the framework of general relativity by solving the Einstein equations for stationary axi-symmetric space-time. In detail, see Refs. 5, 48 and 49 and references therein.

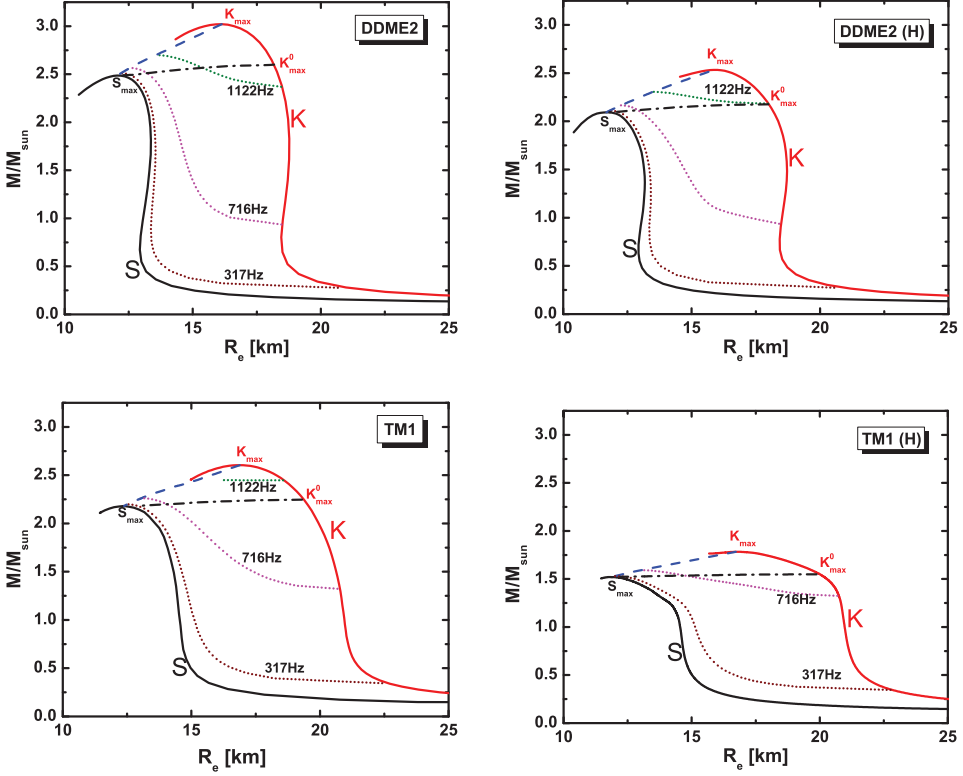


Fig. 1. Left panels: Gravitational mass  $M$  versus equatorial radius  $R_e$  for static and rigidly rotating neutron stars, based on the stiffer DDME2 EOS and the softer TM1 EOS. Right panels: Corresponding ones with hyperons included. Solid line **S**: static models. Solid line **K**: Keplerian (mass-shedding) configurations. The area, bounded by the **S**, **K** curves and a dash line  $S_{\max}K_{\max}$ , consists of points corresponding to stationary rotating configurations. The dash-dot line  $S_{\max}K_{\max}^0$  has the same baryon mass. Three dot lines corresponding to neutron stars rotating stably at  $f = 317, 716$  and  $1122$  Hz, are labeled with rotation frequencies.

In Fig. 1, we show the gravitational mass  $M$  versus equatorial radius  $R_e$  for static and rigidly rotating neutron stars, based on the stiffer DDME2 EOS and the softer TM1 EOS, for both cases with and without hyperons. The lower black solid curve denoted by **S**, is the static limit, i.e. the nonrotating cases. The higher red solid line denoted by **K** is the mass-shedding keplerian limit, where the matter is rotating sufficiently rapidly that the gravitational attraction is not sufficient to keep matter bound to the surface. Three dot lines, corresponding to neutron stars rotating stably at 317, 716 and 1122 Hz, are labeled with rotation frequencies.

The blue dash line  $S_{\max}K_{\max}$  is the onset of instability to radial collapse doomed to collapse into a black hole, as discussed by Cook.<sup>5,50</sup> The area, bounded by the **S**, **K** curves and line  $S_{\max}K_{\max}$ , consists of points corresponding to stationary rotating configurations. The dash-dot line  $S_{\max}K_{\max}^0$  has baryon mass with the maximum allowable baryon mass for nonrotating stars. The configurations

Table 2. Keplerian frequency models for supramassive and normal sequences based on the RMF EOSs, where the values in parentheses denote the results after including hyperons.

	EOS	$M$ ( $M_{\odot}$ )	$R_e$ (km)	$R_p/R_e$	$\varepsilon_c$ ( $10^{15}$ g/cm $^3$ )	$f_K$ ( $10^3$ Hz)	Period (ms)
Supra- massive	DDME2(H)	3.02(2.53)	16.13(15.86)	0.56(0.56)	1.56(1.78)	1.51(1.43)	0.66(0.70)
	PKDD(H)	2.78(2.34)	15.69(15.54)	0.56(0.57)	1.80(1.97)	1.51(1.42)	0.66(0.70)
	TW99(H)	2.48(2.18)	14.16(14.60)	0.57(0.57)	2.24(2.19)	1.67(1.51)	0.60(0.66)
	GL97(H)	2.37(1.87)	14.67(14.84)	0.57(0.58)	2.24(2.43)	1.55(1.37)	0.65(0.73)
	PK1(H)	2.78(1.88)	17.05(16.80)	0.56(0.58)	1.56(1.76)	1.34(1.15)	0.75(0.87)
	TM1(H)	2.60(1.78)	16.88(16.72)	0.57(0.58)	1.61(1.83)	1.32(1.13)	0.76(0.89)
Normal	DDME2(H)	2.49(2.09)	18.35(18.22)	0.54(0.55)	0.79(0.82)	1.16(1.08)	0.86(0.93)
	PKDD(H)	2.33(1.97)	18.44(18.35)	0.56(0.56)	0.86(0.90)	1.11(1.03)	0.90(0.97)
	TW99(H)	2.08(1.83)	16.68(17.00)	0.56(0.56)	1.05(1.00)	1.22(1.12)	0.82(0.90)
	GL97(H)	2.02(1.61)	17.85(18.55)	0.56(0.57)	0.98(0.88)	1.09(0.93)	0.92(1.08)
	PK1(H)	2.32(1.59)	19.66(19.64)	0.55(0.57)	0.73(0.77)	1.01(0.85)	0.99(1.18)
	TM1(H)	2.18(1.52)	19.56(20.18)	0.56(0.57)	0.75(0.68)	0.99(0.80)	1.01(1.26)

below such line can be reached by spinning up a nonrotating star, which are called “normal” configurations. The configurations above such line are so-called “supramassive” configurations. The maximum rotational frequencies  $f_K$  for the “normal” and “supramassive” sequences have been discussed for polytropic and realistic EOSs.<sup>51,52</sup>

To show the results more clearly, we present the keplerian frequency configurations in Table 2 for supramassive sequences and normal sequences based on the RMF EOSs. The informations include the corresponding mass  $M$ , the equatorial radius  $R_e$ , the ratio of polar radius  $R_p$  to  $R_e$ , the central value of the total energy density  $\varepsilon_c$ , the keplerian frequency  $f_K$  and the period  $T$ . It is found that the maximum mass of the neutron star decreases about 16% for DDME2 EOS after including hyperons, and about 30% for TM1 EOS. The values for other EOSs lie in the range of 16–30%. However, the radius and the value of  $R_p/R_e$  change little. For the calculated neutron stars without hyperons, a very short period as small as 0.60 ms ( $f_K = 1.67$  kHz) is possible, and most rapidly rotating “normal” sequences have a period of  $\sim 0.82$  ms. After including hyperons, the allowable  $f_K$  becomes smaller while the corresponding period becomes larger. As shown in Table 2, all RMF EOSs adopted in this paper with and without hyperons could satisfy the restricted condition that the calculated values of  $f_K$  are larger than the maximum observed frequency 1122 Hz.

#### 4. Keplerian Frequency of Neutron Stars

The precisely calculated keplerian frequencies  $f_K$  for a neutron star versus gravitational mass  $M$  based on various RMF EOSs are presented in Fig. 2. To put it another way, these results also represent the minimum mass  $M$  for a certain frequency  $f$ . The predicted relationship between the  $M$  and  $f_K$  can be tested by the future observed data, which can rule out some EOSs.

In Fig. 2, we also show the keplerian frequencies given by the empirical formula

$$f_K(M) = f_0(M) \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{R_S}{10 \text{ km}} \right)^{-3/2} \quad (2)$$

with  $f_0 = 1.08 \text{ kHz}$ , where  $M$  is the gravitational mass of rotating star and  $R_S$  is the radius of the nonrotating star of mass  $M$ . The  $M \sim R_S$  relationship has been shown as the line “S” in Fig. 1. The formula for  $f_K(M)$  is based on a one-to-one correspondence between a static configuration on the **S** boundary, and the rotating configuration with keplerian frequency on the **K** boundary. This correspondence is visualized in Fig. 1. Haensel *et al.*<sup>9</sup> suggested the optimal value of the prefactor  $f_0$  is  $1.08 \text{ kHz}$  for  $0.5M_\odot < M < 0.9M_{\text{max}}^{\text{stat}}$ . Here, we present the results for a larger range  $0 < M < M_{\text{max}}^{\text{stat}}$ . As shown in Fig. 2, it seems that the empirical formula is indeed a good approximation.

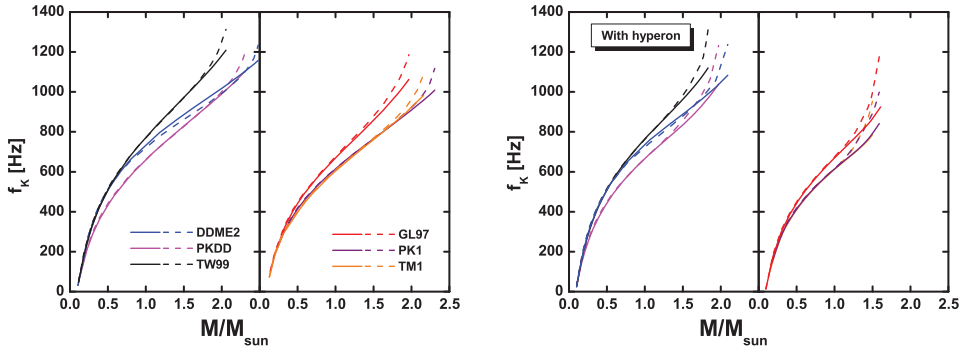


Fig. 2. The precise values of keplerian frequency  $f_K$  calculated with RMF EOSs (solid line) and those calculated using formula  $f_K(M) = 1.08 (M/M_\odot)^{1/2} (R_S/10 \text{ km})^{-3/2} \text{ kHz}$  (dashed line), versus stellar mass  $M$ , for  $0 < M < M_{\text{max}}^{\text{stat}}$ .

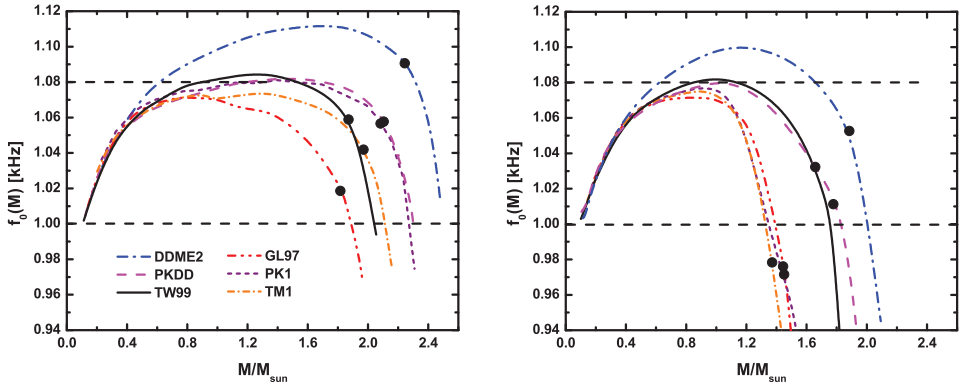


Fig. 3. Precise values of  $f_0(M) = f_K(M)/(M/M_\odot)^{1/2} (R_S/10 \text{ km})^{-3/2}$  based on the calculations adopting RMF EOSs, compared with the empirical values  $f_0 = 1.08 \text{ kHz}$  and  $f_{\text{Roche}} = 1.00 \text{ kHz}$ . The black dots correspond to the results of neutron stars with  $0.9M_{\text{max}}^{\text{stat}}$ .

To be more precise, the validity of prefactor  $f_0 = 1.08$  kHz in the empirical formula is checked in detail in Fig. 3. The value  $f_{\text{Roche}} = 1.00$  kHz from the limit case of relativistic Roche model<sup>9,53</sup> is also plotted in the figure. For the neutron stars without hyperons, the precise values of  $f_0(M)$  is about 1.06 kHz at  $0.5M_\odot$ , while  $f_0(M) \in (1.02, 1.09)$  kHz at  $0.9M_{\text{max}}^{\text{stat}}$ . Thus for  $0.5M_\odot < M < 0.9M_{\text{max}}^{\text{stat}}$ , relative deviations of the empirical formula are typically within 3%, with largest deviation at most 6%. When the mass is close to zero or  $M_{\text{max}}^{\text{stat}}$ , the precise value of  $f_0(M)$  is close to  $f_{\text{Roche}}$ . Extending the mass range to  $0 < M < M_{\text{max}}^{\text{stat}}$ , it is found that the  $f_0(M)$  is the approximately parabolic function of  $M$ , with the values changing from 0.96 kHz to 1.11 kHz. The precise  $f_0(M)$  value is largest for DDME2 while smallest for GL97. Comparing with the DDME2 and GL97 EOSs, the approximation is better for PKDD, TW99, PK1 and TM1 EOSs. In a word, the largest deviation is about 10% for the empirical formula Eq. (2) with  $f_0 = 1.08$  kHz based on the calculated results of  $0 < M < M_{\text{max}}^{\text{stat}}$  with the RMF EOSs.

After including the hyperons, the empirical formula with  $f_0 = 1.08$  kHz becomes worse for large mass region. The largest deviations are at most 10% for  $0.5M_\odot < M < 0.9M_{\text{max}}^{\text{stat}}$ , while 13% for  $0 < M < M_{\text{max}}^{\text{stat}}$ .

Equation (2) provides the simple and universal relationship among the three observable quantities of the neutron stars, i.e.,  $M$ ,  $R$  and  $f_K$ . Next, let us further consider the empirical relationships which are correlative with Eq. (2). As is well known, for a point particle moving in the Schwarzschild space-time around a point (or a spherical) mass  $M$ , the relationship is satisfied as:

$$f_K(M) = \frac{1}{2\pi} \sqrt{\frac{GM}{R_e^3}} \approx 1.833 \text{ kHz} \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{R_e}{10 \text{ km}} \right)^{-3/2}, \quad (3)$$

where  $R_e$  is the equatorial radius corresponding to the mass  $M$  of the neutron star rotating at the keplerian frequency. However, the neutron stars rotating at keplerian frequency become oblate from spherical shape, with  $R_p/R_e \sim 0.56$ . Thus, the precise factor  $f'_0 = f_K(M)/[(M/M_\odot)^{1/2}(R_e/10 \text{ km})^{-3/2}]$  deviates from 1.833 kHz, as shown in Fig. 4. It is found that the  $f'_0(M)$  is a approximately parabolic function of  $M$ , which could be approximately described by a fitting formula  $f'_0(M) = 1.86 - 0.03(M - 1.1)^2$  kHz.

The difference of Eqs. (2) and (3) is that  $R$  respectively represents the radius  $R_S$  of the nonrotating star and the equatorial radius  $R_e$  of the neutron star rotating at keplerian frequency.  $R_S$  and  $R_e$  are functions of mass  $M$  of neutron stars, which are shown as lines ‘‘S’’ and ‘‘K’’ in Fig. 1. The relationship between  $R_e$  and  $R_S$  with the same mass is shown in Fig. 5. It is found that there is a proportional relationship between  $R_e$  and  $R_S$  with the same mass  $M$  within  $0 < M < 0.9M_{\text{max}}^{\text{stat}}$ , namely,

$$R_e(M) = aR_S(M). \quad (4)$$

Actually, for the extreme relativistic Roche model,  $R_e(M)$  is strictly proportional to  $R_S(M)$  with the same  $M$  with  $a = 1.5$ .<sup>9,53</sup> However, the best-fit proportionality factors in the present calculations are smaller than 1.5 of the Roche model,  $a \approx 1.44$ ,

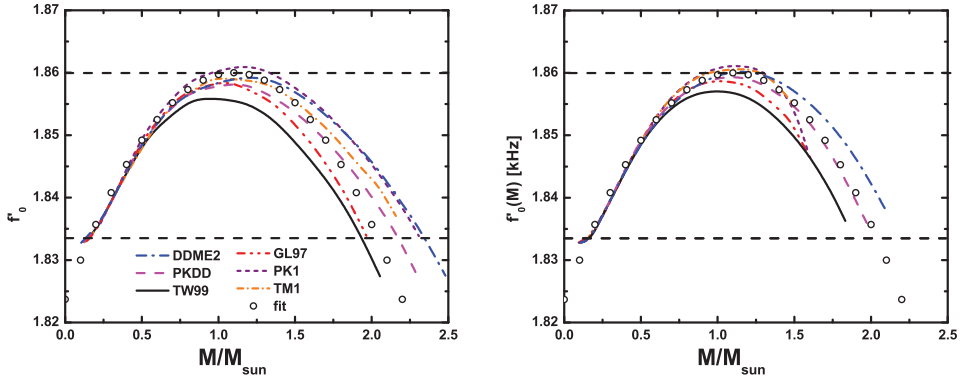


Fig. 4. Precise values of  $f'_0(M) = f_K(M)/(M/M_\odot)^{1/2}(R_e/10 \text{ km})^{-3/2}$  based on the calculations adopting RMF EOSs. The circles denote the values from the fitting formula  $f'_0(M) = 1.86 - 0.03(M - 1.1)^2$ .

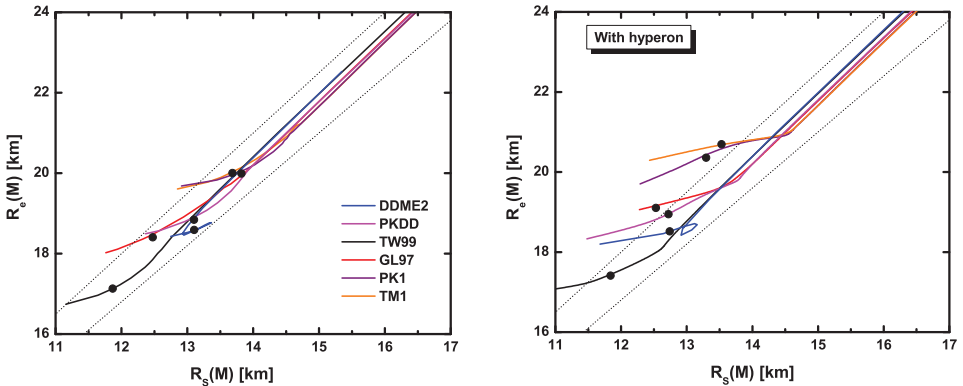


Fig. 5. Equatorial radius of the keplerian configuration  $R_e(M)$  versus circumferential radius of the static configuration  $R_s(M)$  of the same mass  $M$ , for  $0 < M < M_{\text{max}}^{\text{stat}}$ , calculated with the RMF EOSs. Two straight dot lines correspond: upper line to  $a_{\text{Roche}} = 1.5$  and bottom line  $a = 1.4$ . The black dots correspond to the results of neutron stars with  $0.9M_{\text{max}}^{\text{stat}}$ .

which is consistent with the value suggested by Haensel *et al.*<sup>9</sup> For  $M > 0.9M_{\text{max}}^{\text{stat}}$  with the small radius, the proportionality is not so good.

The factors can be connected by  $f_0 = f'_0/a^{3/2}$  according to Eqs. (2) and (3). If  $f'_0 = 1.86 \text{ kHz}$  and  $a = 1.44$  are adopted,  $f_0 = 1.08 \text{ kHz}$  will be given. From the relationships in Eqs. (3), (4) and the variation law of prefactors, we could obtain the empirical formula for keplerian frequency  $f_K$  in Eq. (2) and the variation law of  $f_0$  factor.

## 5. Conclusions

The properties of rapidly rotating neutron stars with and without hyperons have been studied adopting six representative EOSs from the recently developed RMF



theory, including both the nonlinear RMF and DDRMF interactions. The relationships among the observable  $f_K$ ,  $M$  and  $R$  for neutron stars based on various RMF EOSs are presented and analyzed.

We have checked the empirical formula for keplerian frequency  $f_K$  for a star with gravitational mass  $M$ , proposed by Haensel *et al.*<sup>9</sup>  $f_K(M) = f_0(M/M_\odot)^{1/2} (R_S/10 \text{ km})^{-3/2}$  where  $R_S$  is the radius of the nonrotating star. For both the neutron stars with and without hyperons, it is found that the  $f_0$  value is an approximately parabolic function of  $M$  changing from 0.94 kHz to 1.11 kHz for  $0 < M < M_{\text{max}}^{\text{stat}}$ . The ever proposed factor  $f_0 = 1.08 \text{ kHz}$  is found to be an approximation based on the RMF ROSs with the error at most 13%, and such approximation is worse for the neutron stars with hyperons. The present results indicate that the errors should be considered when the empirical formula is used to discuss the properties of neutron stars.

Two facts are correlative with the empirical formula and the variation of  $f_0$ :  $f'_0 = f_K(M)/(M/M_\odot)^{1/2}(R_e/10 \text{ km})^{-3/2}$  is parabolic function with the value near 1.833 kHz; the proportional relationship  $R_e(M) \approx 1.44R_S(M)$ , where  $R_e(M)$  is the equatorial radius of keplerian configuration.

## Acknowledgments

This work was partly supported by the National Natural Science Foundation of China (Grant Nos. 11005069, 11175108, 11205075), the Fundamental Research Funds for the Central Universities (Grant Nos. lzujbky-2012-k07 and lzujbky-2012-7), the Shandong Natural Science Foundation (Grant No. ZR2010AQ005), the Independent Innovation Foundation of Shandong University (Grant No. 2011ZRYQ004) and the Graduate Innovation Foundation of Shandong University at WeiHai (No. yjs12025).

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