A key factor to the spin parameter of uniformly rotating compact stars: crust structure

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Abstract We study the dimensionless spin parameter $j \equiv cJ/(GM^2)$ of different kinds of uniformly rotating compact stars, including traditional neutron stars, hyperonic neutron stars and hybrid stars, based on relativistic mean field theory and the MIT bag model. It is found that $j_{\text{max}} \sim 0.7$, which had been suggested in traditional neutron stars, is sustained for hyperonic neutron stars and hybrid stars with $M > 0.5 M_{\odot}$. Not the interior but rather the crust structure of the stars is a key factor to determine j_{max} for three kinds of selected compact stars. Furthermore, a universal formula $j = 0.63(f/f_{\rm K}) - 0.42(f/f_{\rm K})^2 + 0.48(f/f_{\rm K})^3$ is suggested to determine the spin parameter at any rotational frequency f smaller than the Keplerian frequency $f_{\rm K}$.

Key words: stars: neutron — stars: rotation — spin parameter — equation of state — methods: numerical

1 INTRODUCTION

Compact stars, as one of the most exotic objects in the universe, play the role of a bridge among astrophysics, nuclear physics and particle physics. The interior of compact stars is poorly understood, and may contain a mixture of exotic particles based on the physics of strong interactions such as strangeness-bearing baryons (Glendenning 1985, 1997), condensed mesons (Rho & Wilkinson 1979, Kaplan & Nelson 1986), or even deconfined quarks (Collins & Perry 1975).

In theoretical studies, compact stars can generally be divided into several types by assuming there are different components inside: traditional neutron stars (NSs, composed of β -equilibrium nucleon matter), hyperonic NSs (including strangeness-bearing baryons), hybrid stars (with hadron-quark phase transition) and strange quark stars (composed of up, down and strange quarks) and so on, as shown in figure 1 of Weber et al. (2007). How to identify the type and inner structure of currently known compact stars via observable quantities is always a great challenge.

In past decades, many millisecond pulsars have been reported (Backer et al. 1982, Hessels et al. 2006, Kaaret et al. 2007). Various numerical codes have been developed recently to construct models for rapidly rotating compact stars in general relativity (Stergioulas 2003). As directly measurable quantities, the rotational frequency f of pulsars and its maximum value, namely the Keplerian frequency $f_{\rm K}$, have attracted most of the interest in previous studies of rotating compact stars (Cook et al. 1994, Koranda et al. 1997, Haensel et al. 2009).

Recently, another important characteristic quantity, i.e., the dimensionless spin parameter $j \equiv cJ/(GM^2)$, started to be investigated for rotating compact stars (Lo & Lin 2011), where J is the angular momentum and M is the gravitational mass of the stars. It is suggested that the spin parameter j could play an important role in understanding the observed quasi-periodic oscillations (QPOs) in disk-accreting compact-star systems (Miller et al. 1998, Lattimer & Prakash 2007) and the final fate of the collapse of a rotating compact star (Lo & Lin 2011).

In Lo & Lin (2011), it was revealed that the maximum value of the spin parameter j_{max} of a traditional NS rotating at the Keplerian frequency has an upper bound of about 0.7, which is essentially independent of the mass of the NS as long as the mass is larger than about $1 M_{\odot}$. However, the spin parameter of a quark star described by the MIT bag model does not have such a universal upper bound and could be larger than unity. Thus, the spin parameter extracted from the observations could provide a strict and new constraint on the inner structure of compact stars and the corresponding equation of state (EOS) of dense matter.

Recently, Cipolletta et al. (2015) also confirmed that $j_{\rm max} \sim 0.7$ for a traditional NS. To utilize this constraint in future works, its physical origin, e.g., the reason why $j_{\rm max} \sim 0.7$ for rotating NSs, and the reliability of various types of EOSs for dense matter need further study.

In this paper, the dimensionless spin parameter j of three types of rotating compact stars, namely, traditional NSs, hyperonic NSs and hybrid stars, will be studied using the EOSs from relativistic mean field (RMF) theory (Walecka 1974, Serot & Walecka 1986, Meng et al. 2006) and the MIT bag model (Chodos et al. 1974, Farhi & Jaffe 1984, Witten 1984). The essential factor to determine the maximum value of the spin parameter j_{max} will be investigated.

The paper is organized as follows: In Section 2 we briefly introduce the numerical method and the EOS models used in the calculations. The bulk and characteristic properties of different kinds of compact stars without or with rotation will be analyzed in Section 3. In Section 4, the dimensionless spin parameter $j \equiv cJ/(GM^2)$ of traditional NSs, hyperonic NSs and hybrid stars, as well as the key factor to determine the j_{max} , will be analyzed. Finally a short summary will be given.

2 NUMERICAL METHOD AND EOS MODELS

We numerically model uniformly rotating compact stars with the RNS code (*http://www.gravity.phys.uwm. edu/rns/*), which solves the equation of hydrostationary equilibrium and Einstein's field equations for rigidly rotating stars under the assumption of stationary and axial symmetry about the rotational axis, and reflection symmetry about the equatorial plane. For the details of the model we refer the reader to Stergioulas (2003), Cook et al. (1994), Nozawa et al. (1998), Stergioulas & Friedman (1995) and references therein. The angular momentum J of a rotating compact star is calculated in the code as follows (Stergioulas 2003)

$$J = \int T_{ab} \phi^a \hat{n}^b dV \,, \tag{1}$$

where T_{ab} is the energy-momentum tensor of stellar matter, ϕ^a is the Killing vector in the azimuthal direction that takes advantage of axial symmetry, \hat{n}^b is the unit normal vector field to the t = const. spacelike hypersurfaces, anddV is the proper 3-volume element.

For NSs with hadronic matter, we adopt two typical effective interactions of the density-dependent RMF theory, i.e., TW99 (Typel & Wolter 1999) and PKDD (Long et al. 2004), to model the EOSs without or with hyperons (denoted as TW99N and PKDDN for a traditional NS and TW99H and PKDDH for a hyperonic NS respectively), which have been successfully used to study properties of nuclear matter and NSs (Hofmann et al. 2001, Ban et al. 2004, Sun et al. 2008a, Long et al. 2012, Zhang et al. 2013). For hyperonic NSs, the hyperons Λ , $\Sigma^{\pm,0}$ and $\Xi^{-,0}$ are covered beyond traditional NSs, and the mesonbaryon coupling constants are chosen according to Ban et al. (2004). For hybrid star models, we adopt the MIT bag models (Chodos et al. 1974, Farhi & Jaffe 1984, Witten 1984) with the bag constant B = 90 and 150 MeV fm⁻³ for quark matter and PKDDN from RMF for hadronic matter, which are denoted as Hybrid90 and Hybrid150 respectively (Sun et al. 2008b). In the MIT bag model, we include massless u and d quarks as well as the s quark with mass $m_s = 150$ MeV. The hadron-quark phase transition of hybrid stars is constrained by the Gibbs (Glendenning) construction (Glendenning 1997, 1992).

In several rotation-related phenomena of compact stars such as glitches, the crust structure has been claimed to play an essential role (Lattimer & Prakash 2004, 2007). It is easy to imagine that the matter distribution near a star's surface has a strong influence on the moment of inertia while the star mass is fixed. Thus, the crust structure of a star should play an important role in its angular momentum J. In general, the crust of NSs can be separated into two parts: outer crust and inner crust (Lattimer & Prakash 2004). The main compositions of the outer crust are ions and electrons. The density of the bottom of this part is about $\rho = 4 \times 10^{11} \text{ g cm}^{-3}$.

On the other hand, the neutrons drip out of the nuclei in the inner crust which consists of electrons, free neutrons and neutron-rich nuclei. The density of the interface between the inner crust and core is about $0.5\rho_0$ (i.e., 0.08 fm^{-3}), where ρ_0 is the so called saturation density of nuclear matter. To consider the effects of the crust, two sets of crust EOS are chosen in the low-density region ($\rho < 0.08 \text{ fm}^{-3}$) instead of RMF or MIT calculations: Set 1, the Negele and Vautherin (NV) EOS (Negele & Vautherin 1973) for the inner crust and Baym-Pethick-Sutherland (BPS) EOS (Baym et al. 1971) for the outer crust; Set 2, the Haensel-Pichon (HP) EOS (Haensel & Pichon 1994) for the inner crust and Douchin-Haensel (DH) EOS (Douchin & Haensel 2001) for the outer crust. To investigate the influence of the crust on the rotational properties of compact stars, in the following discussion we will compare the results generated by NV+BPS and HP+DP crust EOSs with those by the "RMF crust EOS," namely, the EOS of β -equilibrium nucleon matter extrapolated from the core to the surface of the stars. It should be noted that, while approaching the lower density region around the star's surface, the NS matter is more energetically favorable to have a configuration of heavy ions sitting on a lattice rather than β -equilibrium nucleons in a liquid phase (Lattimer & Prakash 2004, Weber et al. 2007). Thus, such an assumed "RMF crust EOS" is not a true situation at low densities for NSs or hybrid stars.

In Figure 1, we present the EOSs for different kinds of compact stars, namely PKDDN and TW99N, for the traditional NS, PKDDH and TW99H for the hyperonic NS, as well as Hybrid90 and Hybrid150 for the hybrid stars. Two crust EOSs, namely NV+BPS and HP+DP, are exhibited in the inserted plot to demonstrate a comparison with PKDDN and TW99N, where a logarithmic coordinate is used to show the difference more clearly. The dotted lines in the inserted plot show the boundaries between the inner crust and outer crust, and inner crust and core. It is illustrated that both crust EOSs have a stiffer behavior at very low densities than the "RMF crust EOS" PKDDN and TW99N composed of β -equilibrium nucleon matter.



Fig. 1 The EOSs for compact star matter adopted in the present work, namely the pressure as a function of energy density for traditional NSs (PKDDN and TW99N), hyperonic NSs (PKDDH and TW99H) and hybrid stars (Hybrid90 and Hybrid150). Inserted plot: the adopted EOSs for the crust matter (BPS+NV and HP+DH) at low densities, including the outer crust and inner crust, which are exhibited on a logarithmic coordinate system to compare with PKDDN and TW99N. The dotted lines in the inserted plot show the boundaries between the inner crust and outer crust, and inner crust and core.

Table 1 The gravitational mass $M_{\text{max}}^{\text{sta}}$, the equatorial radius $R_{\text{max}}^{\text{sta}}$ and the central density $\varepsilon_c^{\text{sta}}$ for non-rotating compact stars with maximum mass configuration, and the corresponding values $M_{\text{max}}^{\text{kep}}$, $R_{\text{max}}^{\text{kep}}$ and $\varepsilon_c^{\text{kep}}$ for the Keplerian rotating compact stars with maximum mass configuration. The Keplerian frequency f_K and corresponding spin parameter j_{max} are also given. The values out of/in the parentheses denote the results with NV+BPS/RMF crust EOS.

EOS	M_{\max}^{sta} (M_{\odot})	R_{\max}^{sta} (km)	$(10^{15} \underset{c}{\varepsilon_c^{\text{sta}}} \text{cm}^{-3})$	$\begin{array}{c} M_{\max}^{\text{kep}} \\ (M_{\odot}) \end{array}$	$R_{ m max}^{ m kep}$ (km)	$\overset{\varepsilon_c^{\rm kep}}{(10^{15}{\rm gcm^{-3}})}$	f_K (kHz)	j_{\max}
TW99N	2.08 (2.08)	10.62 (10.63)	2.58 (2.38)	2.48 (2.52)	14.16 (14.24)	2.24 (2.09)	1.67 (1.68)	0.67 (0.70)
PKDDN	2.33 (2.34)	11.78 (11.73)	2.08 (1.98)	2.78 (2.84)	15.69 (15.70)	1.80 (1.72)	1.51 (1.53)	0.67 (0.70)
TW99H	1.83 (1.82)	10.75 (10.97)	2.50 (2.29)	2.18 (2.23)	14.60 (14.78)	2.19 (2.36)	1.51 (1.51)	0.64 (0.68)
PKDDH	1.97 (1.98)	11.48 (11.33)	2.28 (2.35)	2.34 (2.40)	15.54 (15.53)	1.97 (1.85)	1.42 (1.45)	0.64 (0.68)
Hybrid90	1.51 (1.52)	8.99 (8.97)	3.41 (3.26)	1.84 (1.86)	12.36 (12.29)	2.79 (2.84)	1.78 (1.82)	0.67 (0.69)
Hybrid150	1.50 (1.50)	12.29 (11.93)	1.85 (1.79)	1.81 (1.88)	17.13 (16.94)	1.61 (1.46)	1.10 (1.15)	0.63 (0.70)

3 PROPERTIES OF DIFFERENT KINDS OF COMPACT STARS

The bulk properties of compact stars adopting the above EOSs are calculated and presented in Table 1, including the gravitational mass $M_{\text{max}}^{\text{sta}}$, the equatorial radius $R_{\text{max}}^{\text{sta}}$ and the central density $\varepsilon_c^{\text{sta}}$ for non-rotating stars with the maximum allowable mass, and the corresponding values for the Keplerian rotating stars with the maximum allowable mass. The Keplerian frequency f_{K} and corresponding spin parameter j_{max} are also given. It has been checked that NV+BPS and HP+DP crust models give almost the same results, thus only those with NV+BPS/RMF crust are listed out of/in the parentheses. For comparison, Table 2 shows the results for the Keplerian rotating compact stars with the fixed baryonic mass of $M_b = 1.0 M_{\odot}$ and $1.4 M_{\odot}$. One could see these bulk properties sensitively depend on

the chosen high-density EOSs, in both static and Keplerian rotating cases. Taking $M_{\rm max}^{\rm sta}$ as an example, the values change from $1.5 \, M_{\odot}$ to about $2.3 \, M_{\odot}$. By contrast, it is revealed that the influence of the crust on the observable quantities of compact stars is quite small, especially for the cases of maximum allowable mass (see Table 1) where the maximum deviation is found to be 1% for $M_{\rm max}^{\rm sta}$, 3% for $R_{\rm max}^{\rm ta}$, 4% for $M_{\rm max}^{\rm kep}$, 1% for $R_{\rm max}^{\rm kep}$ and 5% for $f_{\rm K}$. The deviation becomes a little larger as the baryonic mass of stars becomes smaller, but still maintains a small value, e.g., for the compact stars with $M_b = 1.0 \, M_{\odot}$ it is about 5% for $R_{1.0}^{\rm kep}$ and 10% for $f_{\rm K}$ (see Table 2). As an exception, the difference in the maximum value of the spin parameter $j_{\rm max}$ due to the crust effects becomes non-negligible when the baryonic mass of stars decreases, namely, 18% (24%) for stars with $M_b = 1.4 \, (1.0) \, M_{\odot}$. Thus, the sensitivity of

Table 2 The equatorial radius R^{kep} , central density $\varepsilon_c^{\text{kep}}$, Keplerian frequency f_K and corresponding spin parameter j_{max} for the Keplerian rotating compact stars with baryonic mass $M_b = 1.0 M_{\odot}$ (left half) and $1.4 M_{\odot}$ (right half). The values out of/in the parentheses denote the results with NV+BPS/RMF crust EOS.

EOS	$R_{1.0}^{ m kep}$ (km)	$\overset{\varepsilon_c^{\rm kep}}{(10^{15}{\rm gcm^{-3}})}$	f_K (kHz)	$j_{ m max}$	$R_{1.4}^{ m kep}$ (km)	$\varepsilon_c^{\mathrm{kep}}$ $(10^{15}\mathrm{g}\mathrm{cm}^{-3})$	f_K (kHz)	$j_{ m max}$
TW99N	17.90 (17.10)	0.54 (0.51)	0.75 (0.83)	0.68 (0.85)	17.64 (17.22)	0.66 (0.62)	0.90 (0.95)	0.68 (0.80)
PKDDN	19.95 (18.92)	0.41 (0.38)	0.64 (0.71)	0.69 (0.91)	19.68 (19.15)	0.50 (0.47)	0.77 (0.82)	0.69 (0.84)
TW99H	18.09 (17.35)	0.52 (0.41)	0.74 (0.81)	0.69 (0.86)	17.78 (17.60)	0.65 (0.59)	0.88 (0.92)	0.69 (0.81)
PKDDH	19.77 (18.82)	0.41 (0.37)	0.65 (0.72)	0.70 (0.92)	19.58 (18.90)	0.51 (0.46)	0.77 (0.82)	0.70 (0.85)
Hybrid90	13.43 (13.61)	0.97 (0.95)	1.12 (1.14)	0.67 (0.74)	13.59 (13.65)	1.14 (1.12)	1.30 (1.32)	0.69 (0.73)
Hybrid150	19.66 (18.83)	0.42 (0.39)	0.66 (0.72)	0.70 (0.90)	19.43 (19.04)	0.54 (0.48)	0.78 (0.83)	0.70 (0.84)

 j_{max} to the crust structure makes it a possible well characterized quantity in understanding the physics related to the surface of compact stars.

As a fundamental quantity to describe rapidly rotating compact stars, the Keplerian frequency $f_{\rm K}$ is presented in Figure 2 at various gravitational masses for traditional NSs (PKDDN and TW99N), hyperonic NSs (PKDDH and TW99H) and hybrid stars (Hybrid90 and Hybrid150). The solid (dashed) lines represent the results with NV+BPS (RMF) crust EOS. It can be found that the effect of crust structure on the relationship of $f_{\rm K} \sim M$ is not strong, in agreement with the above discussion. An approximate empirical formula, $f_{\rm K}(M) =$ $C(M/M_{\odot})^{1/2} (R_S/10 \text{ km})^{-3/2}$, was proposed by combining the Keplerian frequency $f_{\rm K}$, the gravitational mass M and the radius R_S of a non-rotating static star with the same M (Lattimer & Prakash 2004, Haensel et al. 2009). When $0.5~M_{\odot}~<~M~<~0.9 M_{\rm max}^{\rm stat},$ it has been suggested that C = 1.08 kHz for NSs and C = 1.15kHz for quark stars (Haensel et al. 2009). In Zhang et al. (2013), we have systematically checked this empirical relationship based on the various RMF EOSs without and with hyperons, and found it is valid in all of the cases. In the present work, such an empirical relationship with C = 1.08 kHz is also demonstrated to be applicable in hybrid stars. In fact, it is found that the formula $f_{\rm K}(M) =$ $1.08 \text{ kHz}(M/M_{\odot})^{1/2} (R_S/10 \text{ km})^{-3/2}$ is a good approximation for both cases in Figure 2 with NV+BPS and RMF crust EOSs, but the deviation from the model is within just about 2% for $0.5 M_{\odot} < M < 0.9 M_{\text{max}}^{\text{stat}}$.

4 DISCUSSION ON DIMENSIONLESS SPIN PARAMETER

Now we turn to the dimensionless spin parameter $j \equiv cJ/(GM^2)$. Lo & Lin (2011) have shown that $j_{max} \sim 0.7$ for traditional NSs, but the spin parameter of a quark star modeled by the MIT bag model can be larger than unity and does not have a universal upper bound. It is interesting to further study the reliability of the result with various selected EOSs of dense matter, such as hyperonic NSs and hybrid stars, which were not considered in Lo & Lin (2011). The calculated results for strange quark stars with the MIT bag model are the same as those in Lo & Lin

(2011), which have $j_{\rm max} > 1$ for most star masses, thus we do not show them here.

The maximum value of the spin parameter j_{max} of a rotating compact star for traditional NSs, hyperonic NSs and hybrid stars is shown in Figure 3 as a function of the gravitational masses, which are denoted by solid lines. Here we adopt NV+BPS as the crust EOS. It is shown clearly that the values of $j_{\rm max}$ are near 0.7 not only for the traditional NS, but also for the hyperonic NS and hybrid stars. In other words, the j_{max} values lie in the range of 0.65 ~ 0.7 for the large mass region $M > 0.5 M_{\odot}$. We also obtain the same solid conclusions for j_{max} by adopting another crust EOS, DH+HP EOS, which is not presented here. As discussed in Table 1, the bulk properties of M, $f_{\rm K}$ and R sensitively depend on the matter EOS of dense matter. However, traditional NSs, hyperonic NSs and hybrid stars manifest almost the same behavior for $j_{\rm max} \sim 0.7$. It is very interesting to search for the physical origin of such universal results of j_{max} for different kinds of NSs. Obviously, such an origin could not be attributed to the interior of the compact stars.

As mentioned above, the contribution of crust structure is worth analyzing. To clarify the influence of the crust structure on j_{max} , we plot the results with the "RMF crust EOS" as dashed lines in Figure 3. It is apparent in Table 1 that the influence of the crust on the observables of compact stars is quite small. However, j_{max} manifests very different properties after adopting the RMF crust EOSs as seen in Figure 3. The values of j_{max} can be larger than 0.7, mostly located in the range of $0.7 \sim 1.0$, and sensitively depend on star mass. It is clear now that the crust structure around the star's surface is an essential factor to determine the properties of the spin parameter of the compact stars, especially its maximum value. From Figure 3, one also finds that the divergence of j_{max} between the curves with NV+BPS crust and RMF crust EOS decreases gradually as the star mass approaches the maximum value. The effects of the crust on j_{max} clearly decrease with star mass. In particular, it is seen that the value of j_{max} for configurations with maximum mass constructed with the RMF crust EOS reduces to the "universal" value around 0.7.

To explain such mass-dependent behavior, it would be helpful to compare the density profiles of rotating stars constructed with the NV+BPS and RMF crust EOSs.



Fig. 2 The Keplerian frequency $f_{\rm K}$ as a function of the gravitational mass for traditional NSs (PKDDN and TW99N), hyperonic NSs (PKDDH and TW99H) and hybrid stars (Hybrid90 and Hybrid150). The solid (dashed) lines represent the results with NV+BPS (RMF) crust EOS.



Fig. 3 The maximum value of the dimensionless spin parameter j_{max} of a rotating compact star as a function of gravitational mass for traditional NSs (PKDDN and TW99N), hyperonic NSs (PKDDH and TW99H) and hybrid stars (Hybrid90 and Hybrid150). The solid (dashed) lines represent the results with NV+BPS (RMF) crust EOS. The reference value $j_{max} = 0.7$ is shown as well with the dotted line.



Fig.4 The energy density profile of the Keplerian-rotating traditional NSs with 1.0 M_{\odot} (*left panel*) and maximum (*right panel*) baryonic mass configurations, plotted in the plane passing through the rotational axis of the stars. The PKDDN model is used with two crust settings: NV+BPS (*upper panel*) and RMF (*lower panel*) crust EOSs. The dashed lines represent the interface between crust and core. The unit of the energy density is 10^{15} g cm⁻³.



Fig. 5 The dimensionless spin parameter j as a function of the scaled rotational frequency $f/f_{\rm K}$ for traditional NSs (PKDDN and TW99N), hyperonic NSs (PKDDH and TW99H) and hybrid stars (Hybrid90 and Hybrid150). The baryonic mass of the stars is fixed at $M_b = 0.6, 1.0, 1.4$ and $1.8 M_{\odot}$, respectively. The solid (*dashed*) line represents the results calculated with the NV+BPS (RMF) crust EOS. The circles denote the reference values from the fitting formula $j = 0.63(f/f_{\rm K}) - 0.42(f/f_{\rm K})^2 + 0.48(f/f_{\rm K})^3$.

Taking the result from the PKDDN model as an example, we plot the energy density profile of the Keplerian-rotating NSs in the plane passing through the rotational axis of the stars in Figure 4, with two crust settings: NV+BPS and RMF crust EOSs. The results for 1.0 M_{\odot} and maximum baryonic mass configurations are shown in the left and right panels, respectively. It can be clearly seen that the energy density distribution is sensitive to the choice of the crust EOS for a star with 1.0 M_{\odot} . The rotating stars constructed with the RMF crust EOS have larger energy densities around the star's surface, which explicitly enhance the angular momentum and thereby the values of j_{max} . However, the thickness of the crust obviously decreases with increasing baryonic mass so that the angular momentum of the star is mostly contributed by its core, leading to the relatively smaller divergence of the energy density distribution due to the crust structure, in agreement with the properties of j_{max} shown in Figure 3.

Not only the maximum value, but also the results of the spin parameter j at various rotational frequencies could be affected by the crust structure. The spin parameter j as a function of the scaled rotational frequency $f/f_{\rm K}$ for traditional NSs, hyperonic NSs and hybrid stars with NV+BPS (RMF) crust structure is displayed in Figure 5. The baryonic mass of the stars is fixed at $M_b = 0.6, 1.0, 1.4$ and $1.8 M_{\odot}$, respectively. When NV+BPS (or HP+DH) crust is introduced (solid lines), it is surprising that the curves are almost overlapping for all the EOSs and mass sequences, namely independent of the structure of the interior of the compact stars and their mass. No matter which kind of composition is in the interior of the compact stars, such unified relationship remains, which could be fitted approximately by the formula

$$j = 0.63(f/f_{\rm K}) - 0.42(f/f_{\rm K})^2 + 0.48(f/f_{\rm K})^3$$
, (2)

as denoted with circles in Figure 5. Such a universal fitting formula could be used to deduce the third value when we

have obtained two values of spin parameter j, rotational frequency f and Keplerian frequency $f_{\rm K}$.

When the assumed RMF crust EOS is utilized, it is clearly seen that the curves have a quite different tendency compared to the fitting formula. The deviation becomes larger for a smaller mass sequence. Hence, a compact star source discovered with smaller mass would be a better discriminator to study the crust physics around the star's surface. The largest deviation appears at the Keplerian frequency for each fixed mass sequence. The results of the hybrid stars with $M_b = 1.8 M_{\odot}$ cannot start from the nonrotating configuration because they belong to the supramassive sequence (Cook et al. 1994). In short, this figure emphasizes the important role of the crust structure in the spin parameter once again, especially for compact stars with smaller mass. Various other RMF EOSs are also utilized to study the effects of crust on *j*, and it is verified that the results do not sensitively depend on the selections.

From the above discussion, the core contains up to 99% of the mass of the compact stars but the crust contains less than 1% (Lattimer & Prakash 2004). However, it is amazing to find that just this crust structure becomes a key factor to determine the properties of the spin parameter j, particularly its maximum value. Not the interior of the compact stars but rather the crust structure is one of the most important physical reasons for the stability of j_{max} for different kinds of compact stars.

Let us briefly discuss astrophysical implications of the results of the dimensionless spin parameter. How to distinguish a strange quark star from an NS is still currently an unsolved problem (Li et al. 2011, Yu & Xu 2010). It is suggested that the collapse of a uniformly rotating NS must have j > 1 to form a massive disk around the final black hole, which further leads to the black-hole accretion model of gamma-ray bursts (Shibata 2003a,b, Duez et al. 2004, Piran 2004). Thus, in Lo & Lin (2011) it is proposed that the collapse of a rapidly rotating quark star could be

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a possible progenitor for the black-hole accretion model of gamma-ray bursts. From our discussion, this conclusion is still valid, as other kinds of compact stars hardly have j > 1 for $M > 0.5 M_{\odot}$ except for strange quark stars with a bare quark-matter surface.

Another issue is twin-peak QPOs (kHz QPOs), which appear in low-mass X-ray binaries (LMXBs) (Török et al. 2013b). The relationship between M and the spin parameter *i* plays an important role in the understanding of observed QPOs (Török et al. 2013b,a, 2012). In Kato (2008), based on a Resonantly Excited Disk-Oscillation Model, it is suggested that the observed correlations in Cir X-1 could be well described by adopting $M\,=\,1.5\,\sim\,2.0~M_{\odot}$ and $j \sim 0.8$. Thus, the central star in Cir X-1 could be a quark star if Kato's model is assumed to be correct (Lo & Lin 2011), since the calculated uniformly rotating NSs with hadronic matter cannot have j > 0.7. Here, $j_{\rm max} \sim 0.7$ is sustained for traditional NSs, hyperonic NSs and hybrid stars with $M > 0.5 M_{\odot}$ as long as a proper crust EOS is considered, implying the complexity of the inner structure of the central star in Cir X-1.

5 CONCLUSIONS

In conclusion, the dimensionless spin parameter j of uniformly rotating compact stars has been investigated based on various EOSs provided by RMF theory and the MIT bag model. $j_{\rm max} \sim 0.7$, which was suggested in traditional NSs, holds true for hyperonic NSs and hybrid stars when $M > 0.5 M_{\odot}$. Thus, the present results further support j > 0.7 as evidence to identify strange quark stars from other types of compact stars, which is consistent with the argument in Lo & Lin (2011). After comparing with calculated results based on the real crust EOSs and assumed "RMF crust EOS," it is shown that the crust structure is a key factor to determine the properties of the spin parameter of compact stars. Not the interior of the compact stars but rather the crust structure provides the physical origin of the stability of j_{max} for different kinds of compact stars. Furthermore, a universal formula $j = 0.63(f/f_{\rm K}) - 0.42(f/f_{\rm K})^2 + 0.48(f/f_{\rm K})^3$ is suggested to determine the spin parameter at any rotational frequency for the above three kinds of compact stars with $M > 0.5 M_{\odot}$.

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